Fall 2016/MA 314 **HW 2: We have nothing to fear but sphere itself!**

Geometry is a subject full of wonderful problems and interesting methods drawn from all quarters of mathematics. Some of the problems below are just intended to cement basic concepts or give practice with basic calculations, while others are intended to require substantial insight and even, occasionally, cleverness.

Remember that, although you are encouraged to work together, all of your write-ups must be your own (no copying someone else's solution - not even with minor wording changes.) **List the names of everyone you worked with on the HW!** You are encouraged to **not** look online for solutions - your time is better spent wrestling with the proof yourself or getting help from the professor, than squandering it online. Remember to follow the formatting requirements for your Also, the readings for the week are listed below for convenience, but you should look at the reading assignment webpage to see what the day-by-day reading (and reading response) schedule is.

1. Reading

Read the following in this order.

(1) Schwartz: Chapter 9.1 - 9.3

(2) Bonahon: Chapter 3

(3) Bonahon: Chapter 2.1 - 2.5. Also look at appendix T.4 on complex numbers, if necessary.

(4) Schwartz: Chapter 10.1 - 10.6.

(5) Steven Strogatz's blog post on spherical geometry "Think Globally":
http://opinionator.blogs.nytimes.com/2010/03/21/think-globally/?_php=true&_type=blogs&_r=0

(6) Gary Antonick's blog post on hyperbolic geometry "The Non-Euclidean Geometry of Whales":
http://wordplay.blogs.nytimes.com/2012/10/08/whale/

2. To do

(1) Bonahon Exercise 3.2 (we did this in class, but write up your own solution.) In part (a), there is a typo: the Π in the last line of part (a) should be a Π'.

(2) Bonahon Exercise 3.6 a - c (for a hint, see Schwartz section 9.3) The point is to prove that the area of a spherical triangle is the difference between its angle sum and π. You do not necessarily need to precisely follow the way Bonahon has broken the problem up into parts.

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(3) Hyperbolic geometry calculations.

In your reading, the hyperbolic plane is defined as the set

$$\mathbb{H}^2 = \{(x,y) \in \mathbb{R}^2 : y > 0\}$$

with arc length of a parameterized C^1 curve $\gamma(t) = (x(t), y(t))$ for $a \leq t \leq b$ defined by

$$l_{\text{hyp}} = \int_a^b \frac{||\gamma'(t)||}{y(t)}\, dt.$$

(a) Summarize the proof of the result (Lemma 2.1) that l_{hyp} is a path metric.

(b) Sketch a figure in the hyperbolic plane and its image under the standard inversion (Section 2.2.3 of Bonahon)

(c) (Homotheties) Let $\lambda > 0$ be fixed. Let $f(x,y) = (\lambda x, \lambda y)$. Let $\gamma \colon [a,b] \to \mathbb{H}^2$ be a C^1 curve. Prove that γ and $f \circ \gamma$ have the same hyperbolic length.

(d) (Horizontal translations) Let x_0 be fixed. Let $g(x,y) = (x + x_0, y)$. Let $\gamma \colon [a,b] \to \mathbb{H}^2$ be a C^1 curve. Prove that γ and $g \circ \gamma$ have the same hyperbolic length.

(e) Let $Q = [0,1] \times [1,2]$ and $Q' = [0,1] \times [2,3]$ be squares in $\mathbb{H}^2$. Compute their hyperbolic area (for the formula for hyperbolic area, see the end of the Section 10.3 of Schwartz. You'll have to remember how to compute a double integral. Probably you'll need to review Fubini's theorem to remind yourself how to do that.) Comment on the result of your calculations. (Remark: Bonahon Exercise 2.14 also discusses hyperbolic area, but there are a number of typos in that exercise.)

(4) Schwartz: Exercise 4 on page 122.

(5) Given points (x_0, y_0) and (x_1, y_1) in $\mathbb{H}^2$, work out the formula for the hyperbolic distance between them. You may use any facts about hyperbolic geodesics that you like. You might like to use Bonahon's exercise 2.2 for a hint.

(6) Bonahon Exercise 2.9. (This is quite an important result, used in many parts of mathematics.)