F16 MA 274: Pair Review Project

This assignment is to be completed in class on Friday. You **must** email me before Sunday morning a paragraph explaining how far you got, how discussion with your partner went, and which of the problems you would most like to discuss with me.

The point of this assignment is to help you review for the next exam. You do not need to finish this, but you may want to!

NOTE: This problems are (mostly) all from before Chapter 7. You are expected to study and know the major results from Chapter 7 as well, Do each of the following with your assigned partner!

Here is a partial list of some of the more involved theorems we've proved since the last exam. For each of them, do the following in order:

(a) *Without looking at your book or notes*, state what kind of proof is used and list 2 or 3 key steps of the proof.
(b) *Without looking at your book or notes*, write down as a much of a proof as you can.
(c) Check your book or notes to see what steps or key ideas are missing from your proof.
(d) Discuss how you might have been able to figure out those steps or ideas without consulting your notes or book.

(1) A function $f\colon X \to Y$ is a bijection if and only if it has an inverse function $f^{-1}\colon Y \to X$.

(2) If $f\colon \{1,\dots,n\} \to \{1,\dots,n\}$ is a permutation (i.e. a bijection) with $n \geq 2$, then there exist transpositions,
$$\tau_1, \tau_2, \dots, \tau_k$$
such that
$$f = \tau_k \circ \tau_{k-1} \circ \cdots \circ \tau_2 \circ \tau_1.$$
(Recall that a transposition is a permutation which switches exactly two elements of the set.)

(3) Every natural number $n \geq 2$ is a multiple of a prime.

(4) If G is a finite, connected, planar, non-empty graph with $V(G)$ vertices, $E(G)$ edges, and $F(G)$ faces then
$$V(G) - E(G) + F(G) = 2.$$

(5) For each $n \in \mathbb{N}$, let $A_n = \{1,\dots,n\}$. Then if there is a bijection $f\colon A_n \to A_m$, we have $n = m$.

(6) Let $a, b \in \mathbb{N}$. There exist $q, r \in \mathbb{N} \cup \{0\}$ such that $r < a$ and $b = aq + r$.

(7) If $r \in \mathbb{Q}$ and $r > 0$, there exists $a, b \in \mathbb{N}$ such that a and b have no common factor other than ± 1 and $r = a/b$.

(8) A connected, non-empty graph G has contains an euler circuit if and only if every vertex of G has even valence.

(9) Suppose that X is a set and that there is a sequence (a_n) in X with the property that for all $N \in \mathbb{N}$, there exists $m > N$ so that $a_m \notin \{a_1, \dots, a_N\}$. Then there is an injective sequence (x_k) such that the range of (x_k) is equal to the range of (a_n).

(10) If A is an infinite set, then there exists an injective sequence (a_n) in A (equivalently, there exists an injective function $a\colon \mathbb{N} \to A$.)