

### S16 MA 274: Exam 3 Study Questions

You can find solutions to some of these problems on the next page. These questions only pertain to material covered since Exam 2. The final exam is cumulative, so you should also study earlier material.

- (1) Know the definitions on the website. Any other definitions that you need will be given to you.
- (2) When you write a proof, focus on getting the organization clear and correct. If you have to skip some steps or make an assumption that you don't know how to prove, clearly state that that is what you are doing.
- (3) Know the theorems we've proved in class and the more significant theorems from the homework.
- (4) Don't try to memorize proofs. Instead remember the structure of the proof (proof by contradiction, proof of uniqueness, element argument, etc.) and two or three key steps of the proof. Then at the exam recreate the proof.
- (5) At the exam, leave time to write up a nicely written version of each proof. You should have enough time to sketch your ideas out on scratch paper before writing a final version of the proof.
- (6) Study the previous study guides and exams as well as your homework, class notes, and the sections of the text we covered.
- (7) Prove: If  $(x_n)$  is a sequence in a set  $X$  such that for all  $N \in \mathbb{N}$  there exists  $n \geq N$  such that  $x_n \notin \{x_1, \dots, x_N\}$  then  $(x_n)$  has a subsequence  $(x_{n_k})$  which is injective and such that  $\text{range}(x_{n_k}) = \text{range}(x_n)$ .
- (8) Prove: Suppose that  $(x_n)$  is an increasing sequence in  $\mathbb{R}$ . Then either  $(x_n)$  is eventually constant or  $(x_n)$  has a subsequence which is strictly increasing.
- (9) Be able to prove that a sequence  $(s_i)$  in a set  $X$  either has a constant subsequence or has an injective subsequence.
- (10) Be able to prove that an infinite set contains an injective sequence.

- (11) Be able to prove that something is or is not an equivalence relation. For example, if  $\equiv_7$  is defined on  $\mathbb{Z}$  by  $x \equiv_7 y$  if and only if  $x - y$  is a multiple of 7, prove that  $\equiv_7$  is an equivalence relation.
- (12) Be able to prove that if  $\sim$  is an equivalence relation on  $X$  and if  $f$  is a given function with domain  $X/\sim$  then  $f$  is well-defined. For example, if we define  $f: \mathbb{Z}/\sim \rightarrow \mathbb{Z}/\sim$  by  $f([x]) = [2x]$  then  $f$  is well-defined.
- (13) Be able to prove that addition of equivalence classes in  $\mathbb{Z}/\equiv_p$  is well-defined. That is, Define  $x \equiv_p y$  if and only if  $x - y$  is a multiple of  $p$ . Define  $[x] + [y] = [x + y]$ . Prove that  $[x] + [y]$  is well-defined.
- (14) Prove that addition on  $\mathbb{Q}^+ = (\mathbb{N} \times \mathbb{N})/\sim$  is well-defined where pairs  $(x, y) \sim (a, b)$  if and only if  $xb = ya$ .
- (15) Prove that if  $\sim$  is an equivalence relation on  $X$ , then for all  $x, y \in X$ , if  $[x] \cap [y] \neq \emptyset$  then  $[x] = [y]$ .
- (16) Prove that if  $\sim$  is an equivalence relation on  $X$ , then  $x \sim y$  if and only if  $[x] = [y]$ .
- (17) Prove that if  $\sim$  is an equivalence relation on  $X$ , then  $X/\sim$  is a partition of  $X$ .
- (18) State and prove LaGrange's theorem.
- (19) (a new one) Let  $X = \mathcal{P}(\mathbb{R})$  and define  $\sim$  on  $X$  by  $A \sim B$  if and only if there exists a bijection  $f: A \rightarrow B$ . Prove that  $\sim$  is an equivalence relation.
- (20) (a new one) Let  $X$  be a non-empty set and let  $\mathcal{F}$  be the set of bijections of  $X$  to itself (i.e. permutations of  $X$ ). For  $f, g \in \mathcal{F}$  define  $f \sim g$  if and only if there exists a bijection  $h \in \mathcal{F}$  such that

$$f = h^{-1} \circ g \circ h.$$

Prove that  $\sim$  is an equivalence relation.