MA 313: Nominal study guide for Exam 3

Here are some things you should be sure to know for the exam. You should know other things too.

1. Definitions

- all the stuff from exams 1 and 2 (especially the last part of exam 2)
- mean curvature
- Gaussian curvature
- minimal surface
- least area surface
- geodesic
- covariant derivative
- Christoffel symbols (you do not need their formulas)
- Theorema Egregium statement
- principal curvatures and directions
- parallel transport
- Plateau's problem
- mean curvature flow

2. Results

You should know the statements of these results, how to use them, and the key components of their proofs. For those proofs that are very long or involved you will not be asked to recreate the entire proof.

- Be able to give outlines of the proofs of the following theorems. You do not need to be able to give complete proofs and your outlines may be short as long as they include all the important ideas.
  - least area surfaces are minimal (i.e. have mean curvature 0)
  - a curve is a geodesic if and only if it is length minimizing
  - Theorema Egregium (You do not need to be able to do all the symbol manipulations)
  - A surface of Gaussian curvature 0 without level points is ruled.
  - A ruled surface is developable if and only if it has Gaussian curvature equal to 0
  - If S has constant Gaussian curvature $1/R^2$ the parallel surfaces have constant mean curvature $1/2R$. (Prop. 8.5.2 and Cor. 8.5.3)
- Be able to give complete or mostly complete proofs of the following:
  - If X is a unit vector field on a (portion of a) surface S and if γ is a geodesic on S in the domain of X and if $\nabla_\gamma X = 0$ then the angle between γ and X is constant.
  - If γ is a unit speed curve on a surface then it is a geodesic if and only if geodesic curvature is zero.

- The matrix (in standard coordinates) for $\mathscr{W}$ is $\begin{pmatrix} E & F \\ F & G \end{pmatrix}^{-1} \begin{pmatrix} L & M \\ M & N \end{pmatrix}$.
- The absolute value of Gaussian curvature is the limit of the ratio of the area of the image of the Gauss map and the area of a small portion of the surface (Theorem 8.1.6).
- Euler's Theorem (Thm 8.2.4) and the fact that the principal curvatures are the maximum and minimum values of the normal curvatures.