MA 331	Topology	Fall 2014
	MWF 2 - 2:50	Davis 217

Professor:	Scott Taylor
Office Hours:	MF 12 - 12:50 W 12 - 12:50, 3 - 4 TR 1 - 2 *and by appointment!*
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Course Blog:	`http://web.colby.edu/thegeometricviewpoint/`
Prerequisites:	MA 274 or permission of instructor
Texts:	Bert Mendelson *Introduction to Topology*, 3rd edition Herbert Edelsbrunner and John L Harer *Computational Topology, an introduction*

"Point-set topology is a disease from which the human race will soon recover."
– Poincare
"If its just turning the crank, its algebra; but if its got an idea in it, its topology."
– Lefschetz

The Course: Topology is the mathematical discipline which provides the formal context for discussing "continuity" in its broadest possible sense. Topology as a discipline can be subdivided into Point-Set Topology, Algebraic Topology, Geometric Topology, and Differential Topology. Point-set topology provides the axioms, terminology, and basic results for all the other areas of topology, as well as many other parts of mathematics. However, most of the interesting results from topology arise in the other areas. Our goal this semester is to cover as much of point-set topology as is necessary to get to really interesting results and then to explore facets of algebraic, geometric, and differential topology. The course will be thematically tied together by looking at each of these subjects from a computational point of view. Topology is becoming an increasingly important tool in many areas of applied mathematics, computer science, and data analysis. We'll take a look at some of the ways topology is useful in these areas and you'll have the opportunity to explore more on your own by writing an essay for the blog *The Geometric Viewpoint*.

Although we'll explore applications and the computational side of topology, this course is based on theorems and proofs. These provide knowledge and understanding that is as certain as human endeavors can get. The ability to read, understand, and write proofs is a skill transferable to many other technical domains, both inside and outside mathematics. Computers can compute, but what is truly important in mathematics is the ability to convey technical knowledge and its consequences to a variety of audiences. Proofs are a concise, intellectually rigorous, way of communicating with other mathematically savvy people. You will also have the opportunity to convey mathematics by presenting at the board to your classmates and by writing an essay for a general audience.

Such ambitious goals will require energy and perseverence on your part. In return, you will see some very cool mathematics, not often glimpsed at an undergraduate level. I cannot stress enough how important it is that you devote considerable time and thought to this course. You will often be stuck and you will often need to rewrite your proofs. When you do have a proof it may turn out to be incorrect. Such is the nature of mathematics, and topology in particular. For this reason, it is essential that you do the assigned reading (often IN ADVANCE of the class where the topic is covered) and work together. I will be with you in your struggles: if you need help or encouragement: stop by!

Objectives for increasing mathematical maturity:

- Learn the basic language and results of point-set topology.
- Understand how algebra and calculus can be used to prove results in topology.
- Understand how topology is applicable.
- Engage in significant self-teaching of mathematics.
- Effectively communicate mathematics in both professional and informal styles.
- Write proofs in conventional mathematical style

Major Course Content Objectives

- Understand the definition, basic examples, and essential properties of metric spaces.
- Use the axioms of topology to prove theorems from first principles.
- Be able to prove results and construct examples concerning continuous functions, product spaces, quotient spaces, connectedness, and compactness.

- Be able to apply to prove theorems for simplicial complexes and then to extend them to other spaces using the simplicial approximation theorem.
- Understand the following major theorems of topology:
  - The Jordan Curve Theorem
  - The Classification of Surfaces
  - The Invariance of Dimension
  - The Brouwer Fixed Point Theorem
- Use the fundamental group and homology groups to study topological spaces
- Use algorithms to understand topological spaces and their applications.

Attendance: I value your involvement in the class, therefore class attendance is mandatory. Absence for official Colby activities requires prior approval. Absence for religious reasons will be considered excused if the policy in the college catalogue is followed. I reserve the right to take attendance. More than 3 recorded unexcused absences will result in the reduction by 1/3 of the final course grade. Excessive tardiness or early departure may also result in such a reduction.

Computing Resources: You may occasionally want to use the software to complete some homework problems throughout the semester. *Mathematica* is available on many computers at Colby, including the computers in Davis 216. From Colby's fileserver you may also download and install *Mathematica* on your personal computer for use while on campus. You may also wish to make use of *Grapher*, software which comes bundled with every modern Macintosh. (It can be found under, "Utilities".) It is very easy to use and can draw almost every imaginable type of 2 and 3 dimensional graph (including solutions to differential equations and vector fields.) I highly recommend it.

Academic Honesty: All work in this course must be your own and you should always be prepared to explain or defend it. You are encouraged to work with other students on the homework assignments, but you may not copy another student's work. If you make use of online or print material, other than your textbooks, you *must* cite your sources. On the take-home exams you may use your own texts and notes, but may not discuss the exam with others (except the instructor) and may not use other resources, in print or online. On the final exam you may not use any notes or books and may not discuss the exam with anyone except the instructor. Please read the section on Academic Honesty in the catalogue (pg 43).

Classtime: Our class this semester has a very small number of students, so it will be run in a "semi-seminar" style. This means that you should take special ownership of each class period and come to class prepared to explain, question, and discuss the material of the day. Certain homework problems will be marked with a *. You should be prepared to lead the class in a discussion of those problems.

Evaluation: The numerical course grade will be a weighted average of the cumulative grades with weightings as follows:

25%	Homework Problems	5%	Participation
5%	Reading/Colloquium Responses	10%	Blog Essay
15/20%	Exam 1	15/20%	Exam 2
20%	Final Exam		

The maximum of your scores for Exams 1 and 2 will count at 20% of your grade. The minimum of your scores on Exams 1 and 2 will count as 15% of your grade.

Caveat: earning less than 50% on each of the three exams will result in a grade of "F" for the course.

Course letter grades will be assigned (subject to the above caveat) according to the following scale. Any curve will be determined at the end of the course according to the discretion of the instructor. An A+ may be awarded to students who do exceptionally well and who demonstrate an exceptional interest in the course.

93 – 100 % A	90 – 93 % A-	87 – 90 % B+	83 – 87% B
80 – 83 % B-	77 – 80 % C+	73 – 77 % C	70 – 73% C-
67 – 70 % D+	63 – 67 % D	60 – 63 % D-	below 60% F

Homework: Homework is the most important part of this course. It is where you wrestle with new ideas, generate creative solutions, and convey your thoughts to others. You are expected to write complete, careful, and correct proofs. You should start the homework early, some problems will require multiple attempts and careful thought. If you are having substantial difficulty with a particular problem or the entire homework set, you should email me or come to office hours. I am eager to help you! You are encouraged to work with a partner on the homework, but all work should be your own. In other words, you may discuss particular problems but you may not copy someone else's solution. Doing so is an act of academic dishonesty.

Structure and Schedule: Homework will be assigned and collected each class period. It will always be posted on the course webpage. You are responsible for checking the webpage. If no homework assignment is posted, you should refresh the webpage on your browser and, if that doesn't work, email me to let me know. In the special circumstance that there is no new homework, the webpage will make note of that. If you will not be in class on the day that homework is due you should arrange to turn it in at my office or to have a friend bring it to class. Late homework may be penalized, although you are allowed to turn one homework assignment in late

each week. Note, however, that you are still responsible for the problems marked with * on the day in which they are due.

Grading and Rewrites: Problems will be thoroughly graded for both mathematical correctness and completeness and clarity of the proof. Any problem on which you made a significant effort, but received less than full credit may be rewritten and turned in within one week of when it was originally returned to you. Rewrites are graded using the same criteria as the original and you have the potential to earn full credit on such problems.

Because there will be a lot of paper being passed back and forth between us, you must follow these guidelines for homework:

- Each assignment must have your full name, the date it is due, and the assignment number on each page of the assignment.
- Problems must be turned in in the order they are assigned.
- All solutions must be turned in on 3-hole punched paper and placed into the "official" homework binder.
- Each problem must be written on its own 2-sided sheet of paper. You are encouraged not to turn in your first draft of any of these problems.
- Rewrites must be turned in with the original solution. No credit will be given for rewrites without the original attached. It must be clear exactly which problems from which assignments are being rewritten.
- Homework must be very neat. This means: no messy scratchwork, no cramped writing, no huge eraser marks. If these guidelines are not followed you may be penalized. If you are incapable of writing neatly, you should type your solutions. LaTex is the most popular mathematical typesetting software, but you may also use software like *Scientific Word* or *Mathematica*.

Proof Expectations: It is expected that every graded proof be written in conventional mathematical style. Each statement should follow logically and naturally from the previous ones. It is possible for a proof to be substantially correct but not to receive full credit because the writing was not up to standard. The audience you should keep in mind is an imaginary classmate who is not quite as "bright" as yourself. The danger in communicating mathematics is always assuming your audience knows more than they do! This being topology, good illustrations can help you communicate the important ideas in a proof. Be careful, however, that you do not rely on pictures to the detriment of logic. Always ask yourself if there are other possible pictures

and if your argument relies too heavily on the particular picture you have drawn.

Project: Each class member will contribute an essay to the blog "The Geometric Viewpoint". The project will take a fair amount of time and will require independent investigation, exploration and writing. Completing the project will help improve your ability to convey mathematical concepts to non-mathematicians and will enable to encounter geometric ideas that cannot be fit into the regular content of the course.

Participation: I expect that you will volunteer answers to questions posed to the class as a whole and that you will be prepared to make a contribution to the discussion if specifically called upon. You will also be required to present some proofs at the board. You are welcome to be proactive in scheduling when you are to do that. In the event that I call on you specifically, you do not necessarily need to be able to answer the question I ask, but you should be prepared to give a partial answer, an intelligent guess, or to ask a relevant question. I expect that you will have attempted each day's homework prior to class.

Readings and Colloquium Responses: You will often be assigned reading and reading comprehension questions which must be completed before class. You should email your responses to those questions to me by 6 AM on the due date. Additional readings will be assigned over the course of the semester. These may include research articles and blog posts. For each you must turn in answers to the reading questions distributed with the assignment. *Additionally*, you must attend two Mathematics/Statistics Colloquiua (or symposium presentations). For each you need to write a page summarizing and reacting to the lecture. If your academic, athletic, or work schedule does not allow you to attend colloquia, please see me and I will arrange for an alternative.

Exams: There will be two in-class exams and a final exam. Each exam is cumulative, although the final exam is "more cumulative". The in-class exams will test your knowledge of proofs from the reading and presented in class. The final exam is a take-home exam, due during the final exam period, and will require you to write original proofs. Exams will be designed to test your understanding of the course material. You must understand, and communicate your understanding of the material. You may not discuss the exam with other people and you may not use any resource other than your notes and textbooks. Here is the schedule of exams:

(1) Exam 1: Monday, October 6
(2) Exam 2: Monday, November 3
(3) Final Exam Due: by Monday, Dec. 15 at 4:30 PM.

How to succeed in this course:

Topology is an intense 300-level mathematics course. The course, therefore, expects a level of sophistication and mathematical maturity beyond that expected by introductory calculus courses. In particular: you are responsible for your own learning and the professor is responsible for assisting you in your quest. You need to be willing to do computations and you need to be willing to wrestle with ideas. The following advice is meant to provide suggestions on get more out of the class and to improve your grades:

(1) *Do some math everyday.* Really.

    Learning mathematics is a lot like learning to play an instrument, play a sport, or learn a language. You must practice everyday. The daily homework problems are intended to help you in your daily practice. But you should spend additional time each day working on the weekly homework, reading the text, and studying previous material. On average, you should spend 2 - 3 hours studying for each hour spent in class. In our case, that's 6-9 hours per week of homework and studying.

(2) *Participate in class.* Yes, you.

    Asking and answering questions is a great way to stay engaged with the material and verify for yourself that you know what's going on. The more people that participate, the more fun class is. I, and the rest of the class, value your questions and your answers, right or wrong. In fact, giving a wrong answer to a questions is a great way to learn the right answer. Make an effort to connect each class's activities to previous classes. Try to predict where the material will

be going in the future. Take good notes and listen. If you can't do both, make a deal with a buddy: you take notes and they listen one day and the next day you switch.

(3) *Read the textbook.*

The lectures and the textbook will often present slightly different views on the same material. The examples in the text will (for the most part) differ from the example presented in class. As you read the text, relate what the text does to what happened in class where is it the same, where is it different? If it differs, which approach do you like better? What are the pros and cons of each?

(4) *Form a study group.*

Introduce yourself to other people in the class and meet up outside of class to study and work on homework. Be sure that you don't copy answers, but learn from each other and then write the answers on your own. Compare lecture notes to be sure you copied everything correctly. Ask each other questions and explain material to each other. Working in Davis, isn't required but it makes the math/stats department a happening place – it's a great place to meet up with your classmates.

(5) *Write your own problems.*

As you study for the exams, look back at all the examples done in class, in the text, or on homework. Try modifying them to make up your own problems. Try to solve your own problems. What makes your problems easier or more difficult than the ones you've seen before? Feel free to show me what you've done.

(6) *Be curious.* We will be spending a lot of time, exploring the theoretical mathematical ideas. Be curious about why topology works the way it does. Ask "why". Try to find applications of what we're learning. Try to invent your own topological spaces. Schedule time for curiosity. Block off 2 hours every day for Topology. Any time not spent working on homework, spend studying and playing with geometric concepts. Come up with your own research questions. Computers compute; humans ask creative questions.

(7) *Visit me in my office.*

I love working with students and I love to help you understand and appreciate the beautiful world of mathematics. Feel free to drop by, even when it's not my office hours. If I can't chat, I'll let you

know. Ask crazy questions about the course. Ask questions about my research. Tell me about your past math experiences. Tell me about what subjects you love. Let me know when you have a concert or athletic event.

(8) *Spread the studying out over the semester.*

If you do math everyday, as suggested above, you won't have to cram for exams. You'll be able to sleep and, consequently, to think. You'll be happier and more relaxed. You'll have time to write papers for your other classes. You'll have time to appreciate the snow and the spring. You don't need to pull all-nighters.