MA 331 Homework 7: Seeking closure.

1. Reading

(1) Read 3.4 of Mendelson. Note that we took Theorem 4.5 as the definition of closure. Theorem 4.6 (and its proof) is worth remembering.

2. Problems

(1) On page 86 and following of Mendelson, do:
- Problem 2. (Here $C(A)$ means the complement of A)
- Problems 4 and 5.

(2) Suppose that X is a set and that $\mathscr{U} \subset \mathscr{P}(X)$ is a collection of subsets of X with the property that for all $n \in \mathbb{N} \cup \mathbb{N}$, whenever $U_1, U_2, \ldots, U_n \in \mathscr{U}$, then their intersection $U_1 \cap U_2 \cap \ldots \cap U_n \in \mathscr{U}$. (That is, $\mathscr{U}$ is closed under finite unions). Prove that there is a topology $\mathscr{T}$ on X such that $\mathscr{U} \subset \mathscr{T}$ and so that every set in $\mathscr{T}$ is the union of (perhaps infinitely many) sets in $\mathscr{U}$.

(We say that $\mathscr{U}$ is a **basis** for $\mathscr{T}$)

(Hint: X is the empty intersection of the elements of $\mathscr{U}$ and $\varnothing$ is the empty union of the elements of $\mathscr{U}$. So you just need to prove that if you take finitely many sets which are unions of elements of $\mathscr{U}$ then there intersection is also the union of elements of $\mathscr{U}$.)