MA 331 HW 8: Keep an closed mind – study topology!

1. Reading

(1) Skim Section 3.4. Read Sections 3.5, and 3.6 of Mendelson. Pay attention to the definition of continuous function, homeomorphism, and subspace topology, as well as the theorems about those concepts.

(2) Watch this 6 minute video about stereographic projection. It's made by the mathematician/artist who designed the interest panel on our floor.
https://www.youtube.com/watch?v=lbUOScpu0ws

2. Problems

(1) Do problem 3 on page 91 of Mendelson.

(2) Show that the square in $\mathbb{R}^2$:
$$P = \{(x,y) \in \mathbb{R}^2 : \max(|x|,|y|) = 1\}$$
is homeomorphic to the circle:
$$S^1 = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$$
(Hint: to find the homeomorphism, think geometrically, like we did with stereographic projection.)

(3) Do problems 2, 4, 5, 6 on page 96 of Mendelson.

(4) Find a homeomorphism from the interval $(-\pi/2, \pi/2)$ to the interval $(-\infty, \infty) = \mathbb{R}$ (using the euclidean metric for both intervals). (Hint: think about trig functions – you do not have to prove continuity if your function is well-known to be continuous.) Can you find a homeomorphism from the solid square $(-\pi/2, \pi/2) \times (-\pi/2, \pi/2) \subset \mathbb{R}^2$ to $\mathbb{R}^2$ itself? What about from $D^2 = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$ to $\mathbb{R}^2$?