MA 331 HW 24

Suppose that X is a path connected space (perhaps a surface?) and suppose that $\tilde{X}$ is another space such that there is a surjective map

$$p\colon\ \tilde{X} \to X$$

with the property that for each point $x \in X$, $p^{-1}(x) \subset \tilde{X}$ is a discrete subset of $\tilde{X}$, and there is an open set $U \subset X$ with $x \in U$ such that $\tilde{U} = p^{-1}(U) \subset \tilde{X}$ is homeomorphic to $U \times p^{-1}(x)$ and the restriction of p to each $U \times \{\tilde{x}_i\}$ (where the $\tilde{x}_i$ are the points in $p^{-1}(x)$) is a homeomorphism onto U. Let q be a basepoint in X and let $\tilde{q}_0 \in p^{-1}(q)$.

Prove that if $\gamma\colon\ [0,1] \to X$ is a path with $\gamma(0) = \gamma(1) = q$ then there is a path $\tilde{\gamma}\colon\ [0,1] \to \tilde{X}$ such that the following hold:

(1) $\gamma = p \circ \tilde{\gamma}$

(2) $\tilde{\gamma}(0) = \tilde{q}_0$

Furthermore, if $\phi\colon\ [0,1] \to X$ is a path from $\phi(0) = \phi(1) = q$ and if γ is homotopic to ϕ by a basepoint-preserving homotopy, then there is a homotopy $\tilde{F}\colon\ [0,1] \times [0,1] \to \tilde{X}$ such that the following hold for all $s,t \in [0,1]$:

(a) $\tilde{F}(t,0) = \tilde{\gamma}(t)$

(b) $\tilde{F}(t,1) = \tilde{\phi}(t)$

(c) $\tilde{F}(0,s) = \tilde{\gamma}(0)$

(d) $\tilde{F}(1,s) = \tilde{\gamma}(1)$