MA 331 Homework 2: We have a metrick up our sleeve!

1. Reading

Most homework assignments will require you to read about material we have not yet covered in class. Doing this reading will help you gain independence as a mathematician and will also help us make effective use of class time. If you do the reading in advance of class (as you should!) you'll be better prepared to ask questions, have misunderstandings cleared up, and absorb more of the subtleties of the subject.

(1) Read (or skim) Mendelson, Chapter 1. As you read, ask yourself whether or not you've encountered this concept before. Make note of those concepts you haven't seen before. Pay particular attention to sections 1.4 - 1.7 and 1.9. Before next class, **email me** with a list of topics from this chapter that you don't feel comfortable with.

(2) Read Section 2.3 of Mendelson. Pay particular attention to Definition 3.1 and Theorems 3.3 and 3.4, which provide examples of how to use the definition. We will spend time in class discussing these.

2. You think you have problems!

(1) Do problem 4 on page 34 (Section 1.2) of Mendelson

(2) (*) Let (X,d) be a metric space and $\mathbb{X}$ be the set of non-empty finite subsets of X. For $A,B \in \mathbb{X}$, define

$$d^*(A,B) \quad = \quad \min\{z : \forall b \in B\ \exists a \in A \text{ such that } d(a,b) \leq z\}$$

In other words, the "distance" from A to B is the smallest number z such that each point of B is no more than z from a point of A. Another way of phrasing this is: $d^*(A,B)$ is the smallest number such that B is contained in a ball of radius $d^*(A,B)$ around A.

(a) Let (X,d) be $\mathbb{R}^2$ with the euclidean metric. Let:

$$\begin{aligned} A &= \{(0,0),(1,0),(-1,0)\} \\ B &= \{(0,-1),(1,1)\} \end{aligned}$$

Find $d^*(A,B)$ and $d^*(B,A)$.

(b) Prove that for any non-empty (X,d) the function d^* on $\mathbb{X}$ satisfies the triangle inequality. That is, you must show that for all A,B,C

$$d^*(A,C) \leq d^*(A,B) + d^*(B,A).$$

(Hint: Show that for all $c \in C$, there exists $a \in A$ such that $d(a,c) \leq d^*(A,B) + d^*(B,C)$.)

(c) Prove that the function $\mathbf{d}$ defined on $\mathbb{X}$ by

$$\mathbf{d}(A,B) = d^*(A,B) + d^*(B,A)$$

is a metric on $\mathbb{X}$.

(3) (*) Let X be a set with at least two elements. Let d_X be the discrete metric on X. That is, d_X is the metric from 1.2 problem 7 of Mendelson defined by

$$d_X(x,y) = 1 \Leftrightarrow x \neq y \text{ and } d_X(x,x) = 0$$

for all $x, y \in X$.

Let (Y, d_Y) be an metric space and suppose that $f\colon X \to Y$ is a function. Prove that f is continuous.