MA 331 HW 18: Be Simplicial-Minded!

1. Reading

Read Sections III.1 and III.2 of Edelsbrunner and Harer. In III.2 focus on the definition of nerve and the terms surrounding the definition of homotopy equivalent. The problems below are mostly of the reading comprehension sort.

2. Problems

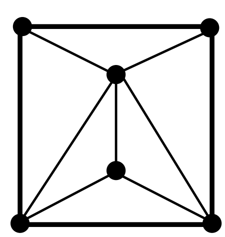
(1) Summarize the difference between an abstract simplicial complex and its geometric realization

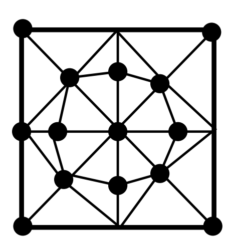
(2) Draw yourself a 2-dimensional connected simplicial complex $|Y|$ with at least 8 triangles and consider a simplicial complex K where $|K|$ is homeomorphic to a circle and there are 4 edges and vertices. Discuss several different kinds of simplicial maps $K \to Y$, drawing pictures to indicate what is going on.

(3) Let K be the simplicial complex pictured on the left and L the simplicial complex on the right. Consider the identity map $\mathrm{id}\colon |K| \to |L|$. With the given simplicial structures, the identity map takes vertices to vertices, but does not take (most) edges to edges. Show how to barycentrically subdivide K n times (for some $n \geq 1$) so that there is a simplicial approximation

$$f\colon \mathrm{Bary}^n(K) \to L,$$

define the simplicial approximation f and explain why it is a simplicial approximation to g. You may find it helpful to label the edges and triangles in order to specify what f does.





(4) Let $|L|$ be the octohedron (so it is homeomorphic to S^2) and let $|K|$ be the circle with the structure of a simplicial complex.

(a) Explain why no simplicial map $|K| \to |L|$ can be onto.

(b) Draw a picture of a curve on $|L|$ which does not pass through any vertices of $|L|$ and then draw a curve which is its simplicial approximation.

(c) Give an informal argument (read "hand-waving") to show that there are maps $\phi : |K| \to |L|$ which require $|K|$ to be barycentrically subdivided arbitrarily many times in order to get a simplicial approximation.

(5) In III.2, the terms retraction and deformation retract are defined. Write a few sentences relating these terms back to the Abrams-Ghrist article.

(6) Draw yourself 5 connected compact sets in $\mathbb{R}^2$ which have non-trivial overlap between various pairs and triples. Explain carefully what the nerve of your collection is.

(7) Suppose that G is a finite connected graph and that G' is obtained from G by removing an edge e with the properties that one of the vertices of e is not adjacent to any other edge of the graph and that the two vertices of e are distinct. Prove that G and G' are homotopy equivalent. (Hint: parameterize e as the interval $[0,1]$ and don't work with the quotient map.)