MA 331 Homework 1: We use metric(s) to measure distance!

Most homework assignments will have both a "Reading" portion and a "Problems" portion. It is essential that you do the reading by the next class. You should also (mostly) do the problems by the next class, although you are allowed to turn one assignment in late per week. The amount of reading and number and difficulty level of problems are designed so that you should, for most assignments, be able to satisfactorily complete the assignment in 2 - 3 hours.

1. Reading

Most homework assignments will require you to read about material we have not yet covered in class. Doing this reading will help you gain independence as a mathematician and will also help us make effective use of class time. If you do the reading in advance of class (as you should!) you'll be better prepared to ask questions, have misunderstandings cleared up, and absorb more of the subtleties of the subject.

(1) Read (don't skim!) Mendelson, Sections 2.1 - 2.2. We'll be spending the next few class periods studying metric spaces in depth.

(2) Read "Todd and Vishal's Blog" on the Platonic Solids. Towards the end of the semester, you'll be asked to write your own blog essay. As you read, work to understand what is being said and also think about how well (or not) the essay is written. There are some blog response questions below.
http://topologicalmusings.wordpress.com/2008/03/01/platonic-solids-and-eulers-formula-for-polyhedra/
Give considered answers to the following and turn them in with the problems below:
Blog Reading Response 1:

  (a) What is a Platonic solid?

  (b) Summarize the proof that there are at most 5 Platonic solids.

  (c) In the essay, they claim that they will prove there are exactly 5 Platonic solids. They don't actually do this, however. What major fact(s) do they need to prove in order to prove their claim?

  (d) Who is the intended audience for the essay?

  (e) React to the quality of writing of the essay (given your idea as to the intended audience): what makes it good? what makes it bad? what could be improved?

(f) Suggest to other possible audiences for an essay on the Platonic solids. How would might the essay be written differently if it were aimed at each of those audiences?

2. Problems

Of course, it is impossible to really understand mathematics without actually doing it! Enjoy these problems concerning metric spaces. (Recall that problems marked with a * will be essential for class discussion, you should spend extra time thinking about and working on those problems.)

(1) Given a metric space (X, d_X), if $Y \subset X$ we can create a metric d_Y on Y by taking the restriction of d_X to Y. That is, for all $y_1, y_2 \in Y$ define

$$d_Y(y_1, y_2) = d(y_1, y_2).$$

We call d_Y the **subspace metric** on Y (thought of as a subspace of X).

- (a) Explain why d_Y is a metric on Y.
- (b) (This one is informal) Let d be the usual (euclidean) metric on $\mathbb{R}^2$. Let $Y = S^1 \subset \mathbb{R}^2$ be the unit circle and let d_Y be the subspace metric on Y. Does or does not d_Y correspond to the distance between two points on Y *as measured* by a being living in Y?

(2) Do problem 7 on page 35 from Section 1.1 of Mendelson.

(3) (*) Let (X, d_X) be a metric space. Let $\mathbb{X}$ be the set whose elements are precisely the non-empty subsets of X having only finitely many elements. For $A, B \in \mathbb{X}$ (so A and B are non-empty subsets of X having finitely many points) define

$$d(A, B) = \min_{a \in A, b \in B} d_X(a, b)$$

(So we calculate $d(A, B)$ by taking a point from A and a point from B finding the distance (in X) between them and then minimizing over all ways of doing this.)

Determine which of the axioms (from Mendelson page 30, Definition 2.1) of a metric space are necessarily satisfied by d and which are not necessarily by d. For those that are satisfied by d, give a proof; for those that are not, give a counter-example. Feel free to draw pictures to assist your explanations.