MA 331 Exam 1 Study Guide

1. Definitions

Know the following definitions:

(1) Metric
(2) continuity for a function between metric spaces
(3) limit of a sequence in a metric space
(4) sequential continuity for a function between metric spaces
(5) open set in a metric space
(6) subspace metric
(7) product metric
(8) topology on a set
(9) open set, closed set in a topological space
(10) neighborhood
(11) limit point of a set
(12) closure of a subset of a topological space
(13) interior of a subset of a topological space
(14) subspace topology
(15) product topology
(16) quotient topology
(17) continuous function between topological spaces
(18) open function between topological spaces
(19) homeomorphism
(20) dense subset of a topological space
(21) basis for a topology
(22) Hausdorff

2. Theorems

Be able to give proofs of the following theorems.

(1) Sperner's Lemma
(2) Brouwer Fixed Point Theorem (assuming Sperner's Lemma)
(3) If (X, d_X) and (Y, d_Y) are metric spaces, then $(X \times Y, d_P)$ is a metric space where $d_P((x_1, y_1), (x_2, y_2)) = \max(d_X(x_1, x_2), d_Y(y_1, y_2))$ is the product metric.
(4) The intersection of topologies on X is a topology on X

(5) If X and Y are metric spaces then $f\colon X \to Y$ is continuous in the metric sense if and only if it is continuous in the topological sense if and only if it is sequentially continuous.

(6) If $f\colon X \to Y$ is a constant function then f is continuous.

(7) The composition of continuous functions is continuous (in topological spaces)

(8) The definitions of the closure of $A \subset X$ as the set of limit points of A and as the smallest closed subset of X containing A are equivalent.

(9) For a subset $A \subset X$ of a topological space there is a largest open set of X contained in A.

(10) The subspace topology is a topology

(11) The product topology is the smallest topology for which all the projection functions are continuous.

(12) The interval $[0,1]$ with the euclidean metric is sequentially compact.

(13) The product of finitely many sequentially compact spaces is sequentially compact.

(14) The image under a continuous function of a sequentially compact space is sequentially compact.

(15) The space $[0,1]/\{0,1\}$ (with the quotient topology) is homeomorphic to the circle $S^1 \subset \mathbb{R}^2$.

(16) If X is an infinite set then $\mathscr{T} = \{U \subset X : U^C \text{ is finite }\}$ is a topology on X.

(17) Prove that if $N \in S^1$ then $S^1 \setminus \{N\}$ is homeomorphic to $\mathbb{R}$.

(18) Let $\sim$ be the equivalence relation on $\mathbb{R}$ given by $x \sim x+n$ for all $n \in \mathbb{Z}$. Then the quotient map $q\colon \mathbb{R} \to \mathbb{R}/\sim$ is open.

(19) The Cantor set is a closed subset of $\mathbb{R}$.