MA 274: Axioms for a Group

A group is a set G with an operation $\circ$ (a way of combining elements of G) such that the following hold:

(G1) For all a and b, $a \circ b$ is an element of G.

(G2) There exists an element $\mathbf{1} \in G$ such that for all $a \in G$,

$$a \circ \mathbf{1} = \mathbf{1} \circ a = a.$$

(G3) For all $a \in G$, there exists $b \in G$ such that

$$a \circ b = b \circ a = \mathbf{1}.$$

(G4) For all a, b, and c in G

$$a \circ (b \circ c) = (a \circ b) \circ c.$$