MA 253 Study Guide 1

1. Definitions

You should know precise definitions for the following terms:

(1) consistent linear system
(2) inconsistent linear system
(3) linear transformation
(4) one-to-one
(5) onto
(6) subspace of $\mathbb{R}^n$
(7) linear combination
(8) span
(9) image
(10) kernel

2. Calculations

You should know how to accurately perform the following kinds of calculations:

(1) Add and scale vectors
(2) Determine if a matrix is in reduced row echelon form.
(3) Row reduce (using Gauss-Jordan) a matrix and/or an augmented matrix
(4) Find the inverse of a matrix using row reduction
(5) Multiply a matrix times a vector
(6) Write the product of a matrix and a vector as the linear combination of the columns of the matrix
(7) Multiply a matrix times another matrix.
(8) Write the product of a matrix and a matrix in terms of the columns of the matrix on the right
(9) Find the matrix which represents a linear transformation.
(10) Find vectors which span the image and kernel of a linear transformation.

3. IDEAS

You should be able to give complete and detailed explanations of the following ideas:

(1) If $A\mathbf{x} = \mathbf{b}$ is a linear system, then either it has a unique solution, or it has no solution, or it has infinitely many solutions.

(2) If $T: \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation, then there is an $m \times n$ matrix A so that $T(\mathbf{x}) = A\mathbf{x}$ for every vector $\mathbf{x} \in \mathbb{R}^n$.

(3) If $T: \mathbb{R}^n \to \mathbb{R}^m$ is linear, one-to-one and onto, then $n = m$.

(4) If $T: \mathbb{R}^n \to \mathbb{R}^m$ is linear, one-to-one, then $n \leq m$

(5) If $T: \mathbb{R}^m \to \mathbb{R}^n$ is linear, onto, then $n \geq m$.

(6) If A is a transition matrix for a mini-web, then the ij entry of A^k is the percentage of people who start at page j and end at page i after following exactly k links.

(7) Explain why an elementary row operation on a matrix can be accomplished by multiplying by an elementary matrix

(8) Explain the geometric effects of linear transformations $\mathbb{R}^2 \to \mathbb{R}^2$ (explained in terms of projections, rotations, reflections, and stretching)

(9) Explain why if A is a 2×2 matrix, then $|det(A)|$ measures how area scales under the linear transformation $T(\mathbf{x}) = A\mathbf{x}$.

(10) Explain why image and kernel are subspaces.