

Hating linear algebra is like hating air! - Daryl Cooper

The Course: Linear algebra is ubiquitous in mathematics, applied mathematics, and the natural and social sciences. It is the subject which allows data to be organized, displayed, and analyzed. It is the subject which underlies the creation and manipulation of images on computers. It is the subject which governs approximation techniques. Linear algebra is like air - it's everywhere and is absolutely necessary for (mathematical) life!
Linear algebra is a collection of tools for understanding linear transformations in both concrete and abstract settings. At its most concrete, it provides tools for effectively solving large numbers of linear equations in lots of variables. In a more abstract setting, linear algebra provides effective means of studying transformations of functions on spaces of any dimension.

Our course begins with the concrete and moves towards the abstract. Along the way we will be concerned with how to compute, why we want to compute, and why the computations work. Although it will be important to do be able to do computations by hand, it is much more important to understand what the computations are telling us and to give precise, intellectually sound, reasons for why things work as they do. This is a class about ideas and completing it successfully will give you the intellectual tools you need to recognize when linear algebra is useful and to apply it in a wide variety of mathematical and scientific settings.

## Objectives for increasing mathematical maturity:

- Think both geometrically and algebraically about systems of linear equations
- Understand aspects of how to apply linear algebra to scientific and mathematical problems
- Understand the relationship between aspect theory and concrete examples
- Engage in significant self-teaching of mathematics.
- Effectively communicate mathematics.


## Major Course Content Objectives

- Be able to do matrix arithmetic by hand and with a computer
- Use matrices to solve systems of linear equations
- Know and use the definitions of linear transformation, linear space, orthogonal transformation, eigenvector and eigenvalue, rank, kernel, basis
- Understand the relationship between matrices, linear transformations, and choice of coordinates.
- Use the Gram-Schmidt process to find orthonormal bases for euclidean spaces
- Understand and use various criteria for determining if a linear transformation is invertible
- Understand the geometric significance of determinants, eigenvalues, and eigenvectors
- Apply linear algebra to solve a variety of applications modelled on scientific problems.


#### Abstract

Attendance: I value your involvement in the class, therefore class attendance is mandatory. Absence for official Colby activities requires prior approval. Absence for religious reasons will be considered excused if the policy in the college catalogue is followed. I reserve the right to take attendance. More than 3 recorded unexcused absences will result in the reduction by $1 / 3$ of the final course grade. Excessive tardiness or early departure may also result in such a reduction.

Computing Resources: On quizzes and exams you may use a graphing calculator. The point of an exam is to test for understanding - such understanding must come through clearly in your answers. It is highly recommended that you use software for matrix arithmetic throughout the semester. Mathematica is available on many computers at Colby, including the computers in Davis 217. From Colby's fileserver you may also download and install


Mathematica on your personal computer for use while on campus. Details will be distributed later.

Upon occasion you may also wish to make use of two other free pieces of software:

- WolframAlpha (wolframalpha.com) is a web-based computational knowledge engine. It can compute most of the integrals and evaluate most of the limits which we will encounter in the course. It can be a bit tricky to use, so you will need think about whatever response it gives you. Another similar tool from the Wolfram people is available at integrals.wolfram.com
- Grapher is software which comes bundled with every modern Macintosh. (It can be found under "Utilities"). It is very easy to use and can draw almost every imaginable type of 2 and 3 dimensional graph. (Including solutions to differential equations and vector fields.) I highly recommend it - it is particularly easy to use.

Note: As you do your homework, distinguish between problems that require you to compute something by hand and those for which a computation is a means to an end. On the former, you should do the computation by hand. On the latter, you may use a computer.

Academic Honesty: All work in this course must be your own and you should always be prepared to explain or defend it. You are encouraged to work with other students on the homework assignments and review exercises, but you may not copy another student's work. If you get help from an online source, be sure to cite it! On exams and quizzes you are not allowed to confer with anybody else, use any notes, books, or online resources, unless otherwise permitted. Calculators are permitted. Please read the section on Academic Honesty in the catalogue (pg 43).

Evaluation: The numerical course grade will be a weighted average of the cumulative grades with weightings as follows:

| $25 \%$ | Homework | $5 \%$ | Reading |
| ---: | :--- | ---: | :--- |
| $10 \%$ | Final Project | $10 \%$ | Quizzes |
| $15 \% / 20 \%$ | Exam 1 | $15 \% / 20 \%$ | Exam 2 |
| $20 \%$ | Final Exam |  |  |

The maximum of your scores for Exams 1 and 2 will count at $20 \%$ of your grade. The minimum of your scores on Exams 1 and 2 will count as $10 \%$ of your grade.

Caveat: earning less than $50 \%$ on each of the three exams will result in a grade of " $F$ " for the course.

Course letter grades will be assigned (subject to the above caveat) according to the following scale. Any curve will be determined at the end of the course according to the discretion of the instructor. An A+ may be awarded to students who do exceptionally well and who demonstrate an exceptional interest in the course.

| $93-100 \%$ | A | $90-93 \%$ | A- | $87-90 \%$ | B+ | $83-87 \%$ | B |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $80-83 \%$ | $\mathrm{~B}-$ | $77-80 \%$ | $\mathrm{C}+$ | $73-77 \%$ | C | $70-73 \%$ | $\mathrm{C}-$ |
| $67-70 \%$ | $\mathrm{D}+$ | $63-67 \%$ | D | $60-63 \%$ | $\mathrm{D}-$ | below $60 \%$ | F |

Homework: Homework is probably the most important part of this course it's when you get to put into practice the concepts you've played with during class. Some of the homework questions may require you to explore some topic which we didn't discuss in class. The purpose of such questions is to help you develop the ability to read and learn mathematics on your own. If you go into a mathematical or scientific career, there will undoubtedly be times when you need to teach yourself some mathematics. If, however, all the homework problems fall into this category, you should check to make sure that you are working on the correct assignment.

Weekly homework will generally be due on Wednesdays and will always be posted on the course webpage. Reading assignments will generally be due every class period and will also be posted to the webpage. You are responsible for checking the webpage. If no homework assignment or reading assignment is posted, you should refresh the webpage on your browser and, if that doesn't work, email me to let me know. In the special circumstance that there is no new homework, the webpage will make note of that. If you will not be in class on the day that homework is due you should arrange to turn it in at my office or to have a friend bring it to class. Late homework may be penalized.
You should start the homework early, some problems will require multiple attempts and careful thought. If you are having substantial difficulty with a particular problem or the entire homework set you should email me or come to office hours. I am eager to help you!

You are encouraged to work with a partner on the homework, but all work should be your own. In other words, you may discuss particular problems but you may not copy someone else's solution. Doing so violates academic honesty. As the course progresses, the question of how much work to show will arise. I encourage you to use common sense. If the work pertains to concepts discussed in class or in the reading, you should show it. If the
work requires substantial effort and thought, you should show it. If the problem is helping you learn how to do a computation, be sure to show all the important steps of the computation. If, however, a computation is simply required in order to do something more interesting, I encourage you to use Mathematica or other software to do the computation. In all cases, you should make the extra effort to explain your work and write something a human can read and understand.

In general, your work is your answer. It is possible for someone to obtain a correct answer but to not receive full credit because their work is incorrect. Conversely, (almost entirely) correct work with an incorrect answer may receive full credit.

Homework must be very neat. This means: no messy scratchwork, no cramped writing, no huge eraser marks. Multiple pages should be stapled and the problems should be in order with section and problem number clearly indicated. If these guidelines are not followed you may be penalized. If you are incapable of writing neatly, you should type your solutions. LaTex is the most popular mathematical typesetting software, but you may also use programs like Scientific Word or Mathematica.

You will frequently have a reading assignment due in class. These assignments will require that you do a certain amount of reading and answer one or two fairly easy questions. Usually the reading will cover material we have not yet or will not discuss in class. I will always assume that you have done the reading by the time that it is due and will not regurgitate in class material you have read on your own.

Homework Extra-Credit: You may earn extra-credit for your homework score by attending colloquia in the Math/Stats department. To receive credit, you must attend the colloquium and then write a short summary and reaction to the talk. Each one that you do will add $0.5 \%$ to your final homework score.

Quizzes: You will be given a quiz almost every Monday. If you are absent on the day a quiz is given, it may be made up within one week of when it was given. You are responsible for requesting a make-up quiz. The lowest quiz score will be dropped from the computation of the course grade.
Projects: In a group of 2-4 people you will complete a substantial project. The content of the project will be related to applications of course material to other scientific disciplines or to other mathematical ideas. The projects will require that you learn the relevant background, extract the most important points, write a summary of your findings and present your findings to the class during the final week of the course. More details will be distributed
later. Successful completion of the project demonstrates your ability to selfteach mathematics and to communicate it effectively to your classmates.

Exams: There will be two in-class exams and a final exam. Each exam is cumulative, although the final exam is "more cumulative". Exams will be designed to test your understanding of the course material, not just your computational abilities. You must understand, and communicate your understanding of the material. Computers, textbooks, notes, and other people may not be used on the exam. The in-class exams will be given on Wednesday, October 1 and Wednesday, November 5. The final exam is currently scheduled for exam period 2 on Wed. Dec. 10 at 1:30 PM. It may not be rescheduled for personal convenience (including airline reservations).

## How to succeed in this course:

Linear Algebra is an intense 200-level mathematics course. The course, therefore, expects a level of sophistication and mathematical maturity beyond that expected by introductory calculus courses. In particular: you are responsible for your own learning and the professor is responsible for assisting you in your quest. In order to truly understand the subject you need to be willing to do computations and you need to be willing to wrestle with ideas. Even more than in calculus, you also need to be willing to take the time to write sound mathematical arguments and detailed explanations. The following advice is meant to provide suggestions on get more out of the class and to improve your grades:
(1) Do some math everyday. Really.

Learning mathematics is a lot like learning to play an instrument, play a sport, or learn a language. You must practice everyday. The daily homework problems are intended to help you in your daily practice. But you should spend additional time each day working on the weekly homework, reading the text, and studying previous material. On average, you should spend 2-3 hours studying for each hour spent in class. In our case, that's 6-9 hours per week of homework and studying.
(2) Participate in class. Yes, you.

Asking and answering questions is a great way to stay engaged with the material and verify for yourself that you know what's going on. The more people that participate, the more fun class is. I, and the rest of the class, value your questions and your answers, right or wrong. In fact, giving a wrong answer to a questions is a great way to learn the right answer. Make an effort to connect each class's
activities to previous classes. Try to predict where the material will be going in the future. Take good notes and listen. If you can't do both, make a deal with a buddy: you take notes and they listen one day and the next day you switch.
(3) Read the textbook.

The lectures and the textbook will often present slightly different views on the same material. The examples in the text will (for the most part) differ from the example presented in class. As you read the text, relate what the text does to what happened in class. Where is it the same, where is it different? If it differs, which approach do you like better- what are the pros and cons of each?
(4) Form a study group.

Introduce yourself to other people in the class and meet up outside of class to study and work on homework. Be sure that you don't copy answers, but learn from each other and then write the answers on your own. Compare lecture notes to be sure you copied everything correctly. Ask each other questions and explain material to each other. The math/stats department is a happening place in the evenings- it's a great place to meet up with your classmates.
(5) Write your own problems.

As you study for the exams, look back at all the examples done in class, in the text, or on homework. Try modifying them to make up your own problems. Try to solve your own problems. What makes your problems easier or more difficult than the ones you've seen before? Feel free to show me what you've done.
(6) Be curious. We will be spending a lot of time, exploring the theory behind Vector Calculus. Try to be curious about why Calculus works. Ask "why" a lot. Feel free to ask about why we're learning something, but also ask about why the ideas came into existence and why the math works one way, but not another. Schedule time for curiousity. Block off 2 hours (most) every day for Linear algebra. Any time that's not spent working on homework or studying, use for playing with the ideas. Use a computer to sketch graphs of vector fields and curves - see if you can predict they way they'll look. Coming up with your own questions is a great way to learn to care about the underlying mechanics of linear algebra. Computers can compute; humans ask creative questions.
(7) Visit me in my office.

I love working with students and I love to help you understand and appreciate the beautiful world of mathematics. Feel free to drop by, even when it's not my office hours. If I can't chat, I'll let you know. Ask crazy questions about the course. Ask questions about my research. Tell me about your past math experiences. Tell me about what subjects you love. Let me know when you have a concert or athletic event.
(8) Spread the studying out over the semester.

If you do math everyday, as suggested above, you won't have to cram for exams. You'll be able to sleep and, consequently, to think. You'll be happier and more relaxed. You'll have time to write papers for your other classes. You'll have time to appreciate the snow and the spring. You don't need to pull all-nighters.
(9) Have an exam strategy.

For the in-class exams, you will have only 50 minutes. Be prepared to do some problems very rapidly and be prepared to think about others. If an example was done in class or on homework, you should be able to repeat it very quickly on the exam. Know what you find difficult and what you find easy. Do the easy things first and then the difficult things. Write something for every problem. If you get stuck, tell me how you'd solve it if you could get unstuck. Figure out what the problem is testing and tell me what you know about that area. If the problem is too hard, rewrite it to make it easier. I love to give partial credit. Give me a reason to give you some. If you find yourself getting nervous: breathe deeply, remind yourself you've studied thoroughly, then figure out how to do the problem. Keep an eye on the time and don't spend too long on any one problem.

