MA 253 Homework Problems 6

Homework has both a "Reading" portion and a "Problems" portion. It is essential that you do the reading by the next class. The reading assignments are posted on a separate webpage. Don't forget to do them!

All page numbers and section numbers refer to the 5th edition of Bretscher's Linear Algebra. Note that most odd numbered problems have solutions in the back of the text. Problems without solutions are worth more points than those with solutions.

Remember to use *Mathematica* to do row operations!

(1) Suppose that $W, V \subset \mathbb{R}^n$ are subspaces of $\mathbb{R}^n$ with $W \subset V$ and that $\dim W = \dim V - 1$. Suppose that $\mathbf{w}_1, \ldots, \mathbf{w}_k$ is a basis for W. Explain why if $\mathbf{v}$ is any vector not in W, then $\mathbf{w}_1, \ldots, \mathbf{w}_k, \mathbf{v}$ is a basis for V.

(2) Explain why every subspace of $\mathbb{R}^n$, other than $\{\mathbf{0}\}$ has a basis.

(3) Section 3.3 (page 131)

   (a) Problems 62, 63, 67, 68

(4) Section 3.4 (page 159)

   (a) Problems 4 - 11 (Remember to use mathematica to do row row reductions, invert matrices, etc.)

   (b) Do parts (a) and (b) of Problems 21 - 24

   (c) Problems 39 - 42, 57, 71

(5) Use change of basis methods to find the matrix (in standard coordinates) for the following linear transformations:

   (a) Rotation by an angle of θ around the line in $\mathbb{R}^3$ spanned by the vector $(2, -2, 7)$.

   (b) The linear transformation $\mathbb{R}^3 \to \mathbb{R}^3$ which is first reflection across the plane with equation $2x + y + z = 0$ and then reflection across the plane $x - y - z = 0$.

   (c) The linear transformation $\mathbb{R}^4 \to \mathbb{R}^4$ which is a reflection across the 3-dimensional subspace defined by the equation $2x + y - z + w = 0$.

   (d) We can think of points of $\mathbb{R}^4$ as points of the form $((a, b), (c, d))$ where $(a, b) \in \mathbb{R}^2$ and $(c, d) \in \mathbb{R}^2$. Let T by the linear transformation of $\mathbb{R}^4$ which reflects points in the first copy of $\mathbb{R}^2$ across the line defined by

$x+y=0$ and which simultaneously rotates the points in the second copy of $\mathbb{R}^2$ by an angle of $\pi/6$ around the origin.

(6) Suppose that $T\colon \mathbb{R}^n \to \mathbb{R}^n$ is a linear transformation such that there is a basis $\mathscr{B} = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$ such that the matrix $[T]_{\mathscr{B}}$ is diagonal. Explain (geometrically) what the linear transformation is doing.

(7) To get some feel for how one might develop intuition for working in $\mathbb{R}^4$ watch the promotional video for a videogame under construction called *Miegakure*. You can find it on Youtube:

https://www.youtube.com/watch?v=9yW–eQaA2It=16

or as the second video (scroll down) on the games website:

http://miegakure.com/.

Another great introduction to thinking about the 4th dimension is the classic novel Flatland (which you can find online or in the library), but you don't have to read that if you don't want to.