MA 253 Homework Problems 4

Homework has both a "Reading" portion and a "Problems" portion. It is essential that you do the reading by the next class. The reading assignments are posted on a separate webpage. Don't forget to do them!

The following problems are due on **Wed. October 1**. All page numbers and section numbers refer to the 5th edition of Bretscher's Linear Algebra. Note that most odd numbered problems have solutions in the back of the text. Problems without solutions are worth more points than those with solutions.

A reminder that you have an exam on Wed. Oct. 1 – study for it!

(1) Section 3.1 (page 85)

  (a) Don't use a computer/calculator for these: 1, 5, 7, 11, 12, 14, 15, 16
  (b) Do these without writing down matrices: 23 - 25
  (c) 30, 34
  (d) 49 (In class we did something similar for the image.)

(2) Section 3.2 (page 130)

  (a) 1 - 3, 6, 8

(3) In class, we claimed that if a linear transformation $T\colon \mathbb{R}^n \to \mathbb{R}^n$ is invertible, then its inverse $T^{-1}\colon \mathbb{R}^n \to \mathbb{R}^n$ is linear. In this problem you'll prove that claim. Throughout, you should expect to use the fact that T is linear. You do not need to (in fact, should not) use anything about matrices for this problem.

  (a) Let $\mathbf{a}$ and $\mathbf{b}$ be vectors in $\mathbb{R}^n$. Show that $T^{-1}(\mathbf{a}+\mathbf{b}) = T^{-1}(\mathbf{a}) + T^{-1}(\mathbf{b})$. (Hint: Use the fact that there are $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ so that $T(\mathbf{x}) = \mathbf{a}$ and $T(\mathbf{y}) = \mathbf{b}$)).
  (b) Show that if $k \in \mathbb{R}$ and if $\mathbf{a} \in \mathbb{R}^n$, then $T^{-1}(k\mathbf{a}) = kT^{-1}(\mathbf{a})$. (Hint: This is a lot like the previous problem)