**MA 253 Homework Problems 2**

*Transforms: More than meets the eye*

Homework has both a "Reading" portion and a "Problems" portion. It is essential that you do the reading by the next class. The reading assignments are posted on a separate webpage. Don't forget to do them!

The following problems are due on **Wed. September 17**. All page numbers and section numbers refer to the 5th edition of Bretscher's Linear Algebra. Note that most odd numbered problems have solutions in the back of the text. Problems without solutions are worth more points than those with solutions.

(1) Section 2.1 (page 53)
4 - 8, 9- 12
13 (this one is very important!)
15
16 - 23 (these problems will help you develop an intuition for how the transforms work)
39 (this one is also crucial!)
43 - 44
50 - 53 (use *Mathematica* as needed)
61

(2) Section 2.2 (page 71)
1 - 11, 16 - 18
19 - 24 (these problems are essential for using linear algebra in geometry)
27, 32, 34, 37, 38

(3) Suppose that the matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ has determinant $k = ad - bc$ and that $k \neq 0$ (see Problem 13 of 2.1). Find the determinant of $A^{-1}$ in terms of $k$.

(4) The problem concerns a variation of linear algebra as you have seen it to date. The results are very useful for understanding continuous maps of a torus to itself, as will be explained in class.

So far in linear algebra, we have used real numbers in our vectors and matrices. Sometimes, however, it is helpful to use *only* integers. (That is the positive and negative whole numbers and 0.) We use $\mathbb{Z}$ to mean the integers (just as $\mathbb{R}$ means the real numbers) and say that we are doing linear algebra *over* $\mathbb{Z}$. This means, for example, that may never use the number $\frac{1}{5}$ in our matrices. Note that the determinant of a $2 \times 2$ matrix with integer entries is also an integer.

(a) Show how to put the following matrix into reduced row echelon form (doing linear algebra over $\mathbb{Z}$, so all intermediate matrices must have integer entries):

$$\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$$

(b) Give an example of a $2\times 2$ matrix with integer entries which cannot be put into reduced row echelon form (doing linear algebra over $\mathbb{Z}$).

(c) Show that if a matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

has an inverse with entries in $\mathbb{Z}$ then the determinant of $A$ is $\pm 1$. (Hint: use problem 3 above)