MA 253 Homework Problems 11

Homework has both a "Reading" portion and a "Problems" portion. It is essential that you do the reading by the next class. The reading assignments are posted on a separate webpage. Don't forget to do them!

Also: Don't forget to work on your projects!

All page numbers and section numbers refer to the 5th edition of Bretscher's Linear Algebra. Note that most odd numbered problems have solutions in the back of the text. Problems without solutions are worth more points than those with solutions.

(1) Section 8.3 (page 411) Problems 1, 6, 7, 8, 11, 15, 17, 18, 19.

(2) Section 4.1 (page 176) Problems 1 - 5, 6 - 10, 12 - 15, .

(3) Let X be a set. Prove that the set of all functions $\{f\colon X \to \mathbb{R}\}$ is a linear space (using a natural notion of addition and scalar multiplication).

(4) Let L be a linear space with basis $v_1, \ldots, v_n$. Let L^* be the set of all linear transformations from L to $\mathbb{R}$. Show that the functions $T_1, \ldots, T_n$ defined by

$$\begin{aligned} T_1(a_1v_1 + \ldots + a_nv_n) &= a_1 \\ T_2(a_1v_1 + \ldots + a_nv_n) &= a_2 \\ &\vdots \quad \vdots \\ T_n(a_1v_1 + \ldots + a_nv_n) &= a_n \end{aligned}$$

give a basis for L^*.

(5) (extra-credit) Let V be a linear space and let $W \subset V$ be a subspace. Declare to elements $u, v \in V$ to be "the same" if $u - v \in W$. We write $u \sim v$.

(a) Show that if $u \sim v$ and $u' \sim v'$ then $u + u' \sim v + v'$.
(b) Show that if $k \in \mathbb{R}$, and if $u \sim v$, then $ku \sim kv$.
(c) Explain (somewhat vaguely if necessary) how we can create a new linear space (denoted V/W) by taking all elements of V which are "the same" and considering them as a single object. (This is a lot like how we consider two numbers x and y to be "the same angle" if $x - y$ is an integer multiple of 2π.)
(Remark: The linear space V/W in the last problem is called a "quotient linear space". They show up in Calculus when you consider two functions to be "the same" when they differ by a constant. In Vector Calculus, they show up when you consider two vector fields to be the same if they differ by a conservative vector field. They play an important role in the mathematical subject known as "homology theory" which (among other things) produces a way of counting "holes".)