MA 253 Homework Problems 10

Homework has both a "Reading" portion and a "Problems" portion. It is essential that you do the reading by the next class. The reading assignments are posted on a separate webpage. Don't forget to do them!

All page numbers and section numbers refer to the 5th edition of Bretscher's Linear Algebra. Note that most odd numbered problems have solutions in the back of the text. Problems without solutions are worth more points than those with solutions.

(1) Apply the Gram-Schmidt process to turn the following vectors into an orthonormal basis for their span. Be sure to show all your work. (The calculations may be unpleasant.)

(a)
$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \end{pmatrix} \right\}$$

(b)
$$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\}$$

(2) Find QR-factorizations of the following matrices (note the connection to the previous problem – you should be able to reuse a lot of work.)

(a)
$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(b)
$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

(3) Explain why every rotation about a line passing through the origin in $\mathbb{R}^3$ is represented by an orthonormal matrix (in standard coordinates).

(4) Suppose that A is an $n \times n$ orthonormal matrix. Give two explanations of why $\det A = \pm 1$: one using the algebraic properties of the determinant, the other using the geometric interpretation of the determinant.

(5) In $\mathbb{R}^6$, let V be the subspace consisting of solutions to both of the following equations:

$$\begin{aligned} x_1 + x_2 - x_3 - x_4 &= 0 \\ x_3 + x_4 - 2x_5 + 3x_6 &= 0 \end{aligned}$$

Find a matrix (in standard coordinates) representing the orthogonal projection $T : \mathbb{R}^6 \to \mathbb{R}^6$ onto V. (Hint: find an orthonormal basis for V and then use the relationship between an orthonormal basis for V and orthonormal projection onto V)