MA 302: HW 3 additional problem

Answer these questions on a separate sheet of paper. Remember that your work must be very neat and complete.

Problem A: Find integrals in 1 variable equal to the lengths of the following curves. You do not need to solve the integrals.

(1) $f(t) = (t, t^2, t^3)$
(2) $g(t) = (\sqrt{t}, \cos t)$

Problem B: Let $\mathbf{x}(t) = (t^3, 2t^3 + 2)$ for $1 \leq t \leq 2$. Reparameterize $\mathbf{x}$ by arc-length.

Problem C: Let $\phi : \mathbb{R} \to \mathbb{R}^2$ be a parameterized curve. In class, we learned how to reparameterized ϕ to become a curve $\phi(s)$ so that for $0 \leq s \leq t$, $\phi(s)$ has length t. Let $k > 0$ be a constant. Show how to reparameterize ϕ to a curve $\phi(u)$ so that for $0 \leq u \leq t$, $\phi(u)$ has length exactly kt.

Problem D: Suppose that at time $t = 0$, a circle of radius ρ is tangent to the parameterized curve $\phi(t)$. The circle rolls along the image of $\phi(t)$ in such a way that at time t, the circle is tangent to $\phi(t)$. Let P be the point on the circle directly to the right of the center of the circle at $t = 0$. In this problem, you will find a parameterization of the path $\mathbf{x}(t)$ taken by P.

(1) Find a parameterization $\mathbf{c}(t)$ for the path taken by the center of the circle. (Hint: use Problem C from HW 2.)
(2) Find coordinates for $\mathbf{x}(t)$ in the tangent space $T_{\mathbf{c}}(t)$.
(3) Find coordinates for $\mathbf{x}(t)$ in the usual coordinate system.
(4) For extra-credit animate the rolling circle and the path traced by P for the curve $\phi(t) = (\cos 3t, \sin 2t)$ for $0 \leq t \leq 2\pi$ and for $\rho = 1/2$.

Problem E: This is the same problem as problem D, but instead of the circle being tangent to the image of $\phi(t)$ at time t, the circle is rolling along the image of $\phi(t)$ so that it makes 1 revolution per second.

(1) At time t seconds, how far along the image of ϕ has the circle rolled?
(2) Find a reparameterization $\psi(t)$ of $\phi(t)$ so that at time t, the circle is tangent to the point $\psi(t)$. (Hint: Use problem C above)
(3) Find coordinates for $\mathbf{x}(t)$ in the usual coordinate system. (Your answer will likely have an inverse function in the expression – you won't be able to get it in closed form.)