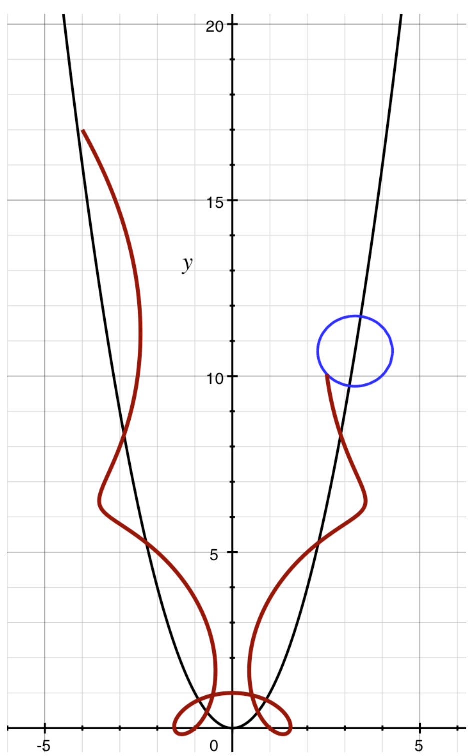
MA 302: HW 2 additional problems

Answer these questions on a separate sheet of paper. Remember that your work must be very neat and complete.

Problem A: Suppose that a circle of radius 1 is rolling down a hill such that the center of the circle is always on the graph of the parabola $y = x^2$. The circle rolls in such that the center of the circle is at the point (t, t^2) at time t and it completes 1 clockwise rotation every 2 seconds. At time $t = 0$, the center of the circle is at the point $(0, 0)$. Let P be the point on the circle directly above the center of the circle at time $t = 0$. Find the parameterization of the path $\mathbf{x}(t)$ taken by the point P as the circle rolls down the parabola. For extra credit use Grapher to make an animation of the circle rolling down the parabola and the path taken by P. You should email the grapher file to me. An example of a still from your animation might be:



Problem B: Suppose that $y = f(x)$ is a differentiable function. Suppose that a circle of radius 1 is resting on the graph of $y = f(x)$ so that the graph of $y = f(x)$ is tangent to the circle at a point (x_0, y_0). Find a formula (in terms of f, x_0, and y_0) for the center of the circle. (Hint: remember that a line tangent to a circle is at right angles to the radius of the circle.)

Problem C: Suppose that at time t a circle of radius ρ is tangent to the parameterized curve $\phi(t)$ where $\phi : \mathbb{R} \to \mathbb{R}^2$. What is a parameterization for the path taken by the center of the circle? (There are two possible answers, depending on which side of the curve the circle lies.)