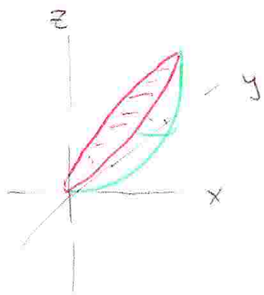
EXAMPLE

COMPUTE VOLUME OF REGION R BOUNDED BY THE GRAPHS OF $z = x$ AND $z = x^2 + y^2$



NOTE THE GRAPHS INTERSECT AT THE CURVE $\begin{cases} z = x \\ z = z^2 + y^2 \end{cases}$

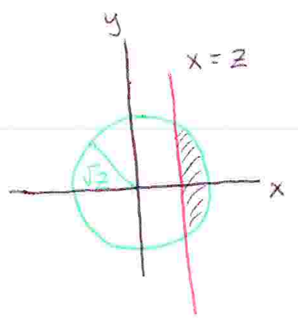
$\Rightarrow \quad 0 \leq z \leq 1$

AND $\quad 0 \leq x \leq 1$

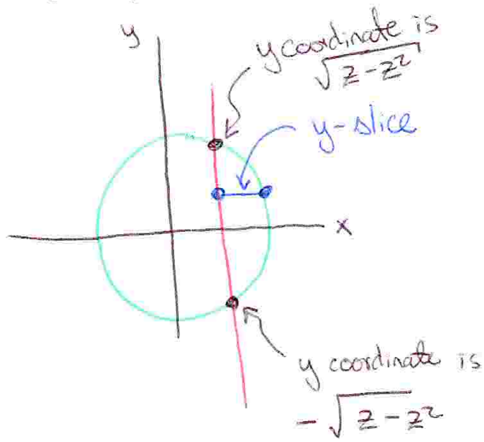
USING LEVEL SETS

FIX z AND COMPUTE AREA OF z-slice

THEN $\underbrace{\iiint_R 1 \, dV}_{\text{VOLUME OF } R} = \int_0^1 A(z) \, dz$



TO FIND $A(z)$, INTEGRATE LENGTHS OF SLICES



$$A(z) = \int_{-\sqrt{z-z^2}}^{\sqrt{z-z^2}} L(y) \, dy$$

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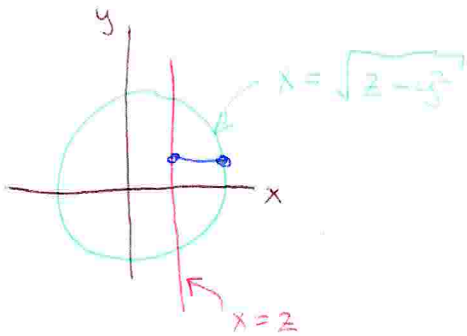
TO FIND $L(y)$:

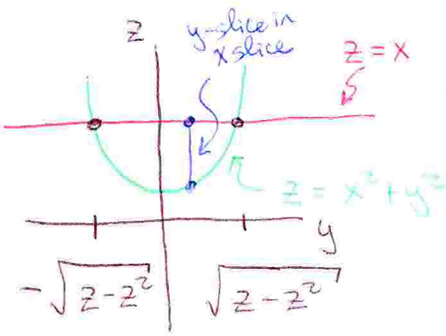
$$L(y) = \sqrt{z - y^2} - z = \int_{z}^{\sqrt{z-y^2}} 1 \, dx$$



THUS,

$$\iiint_R 1 \, dV = \int_0^1 \int_{-\sqrt{z-z^2}}^{\sqrt{z-z^2}} \int_{z}^{\sqrt{z-y^2}} 1 \quad dx \, dy \, dz$$

USING X-SLICES:



$0 \le x \le 1$

$-\sqrt{z - z^2} \le y \le \sqrt{z - z^2}$

$x^2 + y^2 \le z \le x$

$$\Rightarrow \quad \iiint_R 1 \, dV = \int_0^1 \int_{-\sqrt{z-z^2}}^{\sqrt{z-z^2}} \int_{x^2+y^2}^{x} 1 \quad dz \, dy \, dx$$