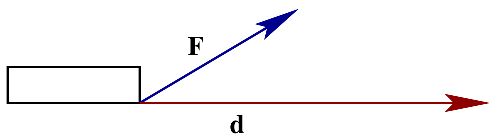
MA 122: Weekly HW 7

Answer these questions on a separate sheet of paper. Remember that your work must be very neat and complete.

Problem 1: If a force of constant magnitude f is used to push an object in the direction of the force a distance d, the work done is $W = fd$. If, however, the force is pushing an object in a direction different from the direction of motion (see the figure below) the work done is defined to be $W = \mathbf{F} \cdot \mathbf{d}$ where $\mathbf{F}$ is the force vector and $\mathbf{d}$ is the vector indicating the direction and distance of motion.



Suppose that a wagon is pulled a distance of 100 m along a horizontal path by a constant force of 50 N. The handle of the wagon is held at an angle of 30° above the horizontal. How much work is done?

Problem 2: Draw a picture of and calculate the volume of the parallelpiped defined by the vectors $\mathbf{v} = (3,0,0)$, $\mathbf{w} = (2,4,0)$ and $\mathbf{u} = (1,3,1)$.

Problem 3: Find the formula for the plane containing the points $(-1,1,1)$, $(1,-1,1)$ and $(2,1,-1)$.

Problem 4: Let $f(x,y) = \frac{x^3 y}{x^6+y^2}$. Show that $\lim_{\mathbf{x}\to\mathbf{0}} f(x,y)$ does not exist.

Problem 5: Let

$$f(x,y) = \begin{cases} xy/(x^2+xy+y^2) & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Explain why f is continuous at all $(x,y) \neq 0$ but discontinuous at $(x,y) = (0,0)$.

Problem 6: Find the ∂/∂_x and ∂/∂_y of the following functions:

(1) $f(x,y) = \sqrt{x^2+y^2}$
(2) $f(x,y) = \ln(x+\sqrt{x^2+y^2})$
(3) $f(x,y) = (x-y)/(x+y)$
(4) $f(x,y) = \int_x^y \cos(t^2)\,dt$ (Hint: Use the fundamental theorem of calculus)

Problem 7: Let $f(x,y) = x^2y^3 + x$.

(1) Find $f_x(1,2)$
(2) Find $f_y(1,2)$
(3) Find $f_{xx}(1,2)$
(4) Find $f_{xy}(1,2)$
(5) Find $f_{yx}(1,2)$
(6) Find $f_{yy}(1,2)$

Problem 8: Let $f(x,y) = \sqrt{1-(x^2+y^2)}$. Let $\mathbf{v} = (v_1, v_2)$ be a point in the domain of f.

(1) What is the graph of $f(x,y)$?
(2) Find $f_x(\mathbf{v})$ and find a point $\mathbf{u}$ on the line through the origin in the xz plane with slope $f_x(\mathbf{v})$.
(3) Find $f_y(\mathbf{v})$ and find a point $\mathbf{w}$ on the line through the origin in the yz plane with slope $f_y(\mathbf{v})$.
(4) Find the equation of the plane containing $\mathbf{0}$, $\mathbf{u}$ and $\mathbf{w}$.
(5) Show that $(v_1, v_2, f(\mathbf{v}))$ is a normal vector for the plane.
(6) Find the equation of the tangent plane to $f(x,y)$ at $\mathbf{v}$.