MA 122: Weekly HW 6

Answer these questions on a separate sheet of paper. Remember that your work must be very neat and complete.

Problem 1: Consider the graph in $\mathbb{R}^4$ of the function $f\colon \mathbb{R}^3 \to \mathbb{R}$ defined by

$$f(x,y,z) = \sqrt{x^2+y^2+z^2}$$

(1) Describe the x, y, and z–slices.
(2) Describe the level sets.
(3) This object in 4–dimensions is an analogue of well-known objects in lower dimensions – what objects?

Problem 2: Solve the following problems (all are taken from the 5th edition of your text pg 700– the odd numbered solutions should be in the back.)

(11) The velocity of the current in a river is $\mathbf{c} = (0.6, 0.8)$ km/hr. The boat moves relative to the water with velocity $\mathbf{v} = (8,0)$. What is the speed of the boat relative to the riverbed? What angle does the velocity of the boat relative to riverbed make with the vector $\mathbf{v}$. Explain in practical terms.
(18) A partical moving with speed v hits a barrier at an angle of $60°$ and bounces off at an angle of $60°$ in the opposite direction with speed reduced by 20%. Find the velocity vector of the object after impact (relative to the point of impact.)
(21) Two forces, represented by the vectors $\mathbf{F}_1 = (8,-6)$ and $\mathbf{F}_2 = (3,2)$ are acting on an object. Give a vector representing the force that must be applied to the object if it is to remain stationary.

Problem 3: Find the angle between the vectors:

(1) $(-1,1,2)$ and $(2,-3,1)$
(2) $(0,1,3)$ and $(3,3,-1)$

Problem 4:

(1) Find the equation of the plane perpendicular to the vector $(-2,3,-1)$ and which contains the point $(1,2,3)$.
(2) Find a normal vector to the plane $2x+y-z=23$.

(3) Consider the point $\mathbf{v} = (1,0,0)$ and the plane P defined by $2x - y + 3z = 0$. What is the shortest distance between the point $\mathbf{v}$ and the plane P?

Problem 5: If $\mathbf{v}, \mathbf{w} \in \mathbb{R}^3$ both lie in the xy plane, what can you say about $\mathbf{v} \times \mathbf{w}$? Explain.

Problem 6: The circle S^1 is the set of all points $(x,y) \in \mathbb{R}^2$ such that

$$x^2 + y^2 = 1.$$

(1) Let $\mathbf{v} = (v_1, v_2)$ be an arbitrary point on S^1 with $v_2 > 0$. Recall that this means that $v_2 = \sqrt{1 - v_1^2}$. Use the derivative of $f(x) = \sqrt{1 - x^2}$ to explain why the equation of the line tangent to S^1 at the point (v_1, v_2) is:

$$y = -\frac{v_1}{v_2}(x - v_1) + v_2.$$

(2) Find a vector $\mathbf{w}$ such that $\mathbf{w}$ is parallel to the line from (a) and has magnitude 1. (Hint: find the equation of the line which is parallel to the line from (a) and which contains $(0,0)$. Pick a point on that line which has distance 1 from the origin.)
(3) Show that $\mathbf{v}$ and $\mathbf{w}$ are perpindicular using the dot product.

Problem 7: The sphere S^2 is the set of all points $(x,y,z) \in \mathbb{R}^3$ such that

$$x^2 + y^2 + z^2 = 1.$$

(1) Suppose that $\mathbf{v} = (v_1, v_2, v_3)$ is an arbitrary point on the sphere S^2.
(2) Find the equation of the plane perpendicular to $\mathbf{v}$ which contains $\mathbf{v}$.
(3) Suppose that $v_3 = 0$ so that $\mathbf{v}$ lies on the xy-plane. Also assume that $v_2 > 0$. Show that the intersection of the plane you found in (2) with the xyplane is the line you found in poblem 6 part 1. Explain the significance.
(4) Do you think it likely that the plane you found in (2) is the tangent plane to S^2 containing $\mathbf{v}$? Explain.