MA 122: Weekly HW 5

Answer these questions on a separate sheet of paper. Remember that your work must be very neat and complete.

Problem 1: Recall that:

- $f(x) = e^x$ is a solution to the initial value problem: $f'(x) = f(x)$ and $f(0) = 1$.
- $f(x) = \cosh(x)$ is a solution to the intial value problem: $f''(x) = f(x)$ and $f(0) = 1$, $f'(0) = 0$
- $f(x) = \cos(x)$ is a solution to the intial value problem: $f''''(x) = f(x)$ and $f(0) = 1$, $f'(0) = 0$, $f''(0) = -1$.

This list raises the question: what functions are there own third derivatives?

Consider the following intial value problem where $a, b, c \in \mathbb{R}$:

$$\begin{aligned} f'''(x) &= f(x) \\ f(0) &= a \\ f'(0) &= b \\ f''(0) &= c \end{aligned}$$

(1) Let $a = 0$, $b = 0$, and $c = 2$. Find a series solution to the initial value problem. (You should compute enough of the coefficients of the series (at least 8) so that it is clear you know how to do it. You do not need to find a general formula for the coefficients unless you want to. If you do find a formula, I'll give you extra-credit.)

(2) Let $a = 0$, $b = 0$, and $c = 0$. Find a series solution to the initial value problem. What function is this? How could you have guessed this solution prior to doing any work?

(3) Let $a = 1$, $b = 1$, and $c = 1$. Find a series solution to the initial value problem. What function is this? How could you have guessed this solution prior to doing any work?

Problem 2: Let $f(x) = \sin(x^2)$.

(1) Use Mathematica to plot $f(x)$ for $x \in (-10, 10)$.

(2) Find a series which represents $f(x)$ on $(-\infty, \infty)$. Be sure you explain how you know that the series you write down converges to $f(x)$ for all values of $x \in \mathbb{R}$.

(3) Use the symmetry of the graph of $f(x)$ to explain why the series has only even terms. (Hint: what does the substitution $x \to -x$ do to both the graph and the series?)
(4) Find a series which represents $f'(x) = 2x\cos(x^2)$ on $(-\infty, \infty)$. Be sure you explain how you know that the series you write down converges to $f'(x)$ for all values of $x \in \mathbb{R}$.
(5) Use the symmetry of the graph of $f'(x)$ to explain why the series you found has only odd terms. (Hint: what does the substitution $x \to -x$ do to both the graph and the series?)
(6) Let $g(x) = \int_0^x f(t)\,dt$. Find a series which represents $g(x)$ on $(-\infty, \infty)$. Be sure you explain how you know that the series you write down converges to $f(x)$ for all values of $x \in \mathbb{R}$.
(7) Use Mathematica to graph the series that you found in part (6) for $x \in (-4, 4)$. How could you have predicted the symmetry of the graph using the series that you found? (Note: It will take Mathematica a while to plot this graph. Be patient.)

Problem 3: In the 5th edition of the Multivariable Calculus book do problem 15 on page 650. You should do this without using a computer or calculator, although you may check your answer. For each part describe one or two features of the graph which made you match it with the equation that you did.

Problem 4: For each of the following equations, sketch a graph of the surface without using a calculator or computer. Briefly describe each surface in words.

(1) $x^2 + y^2 + z^2 = 9$
(2) $x^2 + z^2 = 4$
(3) $z = y^2$.