MA 122: Weekly HW 2

Answer these questions on a separate sheet of paper. Remember that your work must be very neat and complete.

Problem 1: Let $f(x) = \ln(1+x)$ and let $P_n(x)$ be the nth Maclaurin polynomial for $f(x)$. In class we showed that $|E_n(x)| \to 0$ if $x \in [0,1]$. Explain why the proof doesn't show that $|E_n(2)| \to 0$.

Problem 2:

(a) Find an upper bound for the error of the nth MacLaurin approximation $P_n(x)$ to $f(x) = e^x$ for $x \in [0,1]$. (Hint: to make life easy for yourself, at some point you may wish to round e up to 3.)
(b) Find a value of n so that $|E_n(1)| = |f(1) - P_n(1)| \leq .001$. (You should do this using techniques discussed in class: don't just use trial and error.)
(c) Use Mathematica to calculate both $f(1)$ and $P_n(1)$ (for the n from part (b)). What is their difference? Your answer here depends on your answer from (b): different people may have different answers. Hint: To define $P_n(x)$ in Mathematica, use summation notation.
(d) What happens to $|E_n(1)|$ as $n \to \infty$? Explain.

Problem 3: Carefully prove (using the ε definition of limit) that the sequence (a_n) with $a_n = \frac{1}{n^2}$ converges to 0.

Problem 4: Use any of the tools discussed in class to show that the sequence (a_n) with $a_n = \frac{1}{n^2+5}$ converges to 0.

Problem 5: Carefully explain why the sequence $\left((-1)^n\right)$ does not converge.

Problem 6: Let $s_n = \sum_{i=1}^{n}(-1)^i$. Does the sequence (s_n) converge or diverge? Does the series $\sum_{i=1}^{\infty}(-1)^n$ converge or diverge? Explain.

Problem 7: Let $a_1 = 1$ and for $n \geq 2$, let $a_n = \sqrt{1 + a_{n-1}}$. Write out the first 5 terms of the sequence (a_n). Assuming that $\lim_{n\to\infty}(a_n)$ exists, find it.

Problem 8: Let (a_n) be the sequence from problem 8. Prove that $\lim_{n\to\infty} a_n$ exists. (Hint: show that (a_n) is an increasing sequence bounded above by 2.)

Problem 9: Explain why the sequence (s_n) defined by:

$$s_n = \sum_{i=1}^{n} \frac{1}{i!}.$$

converges.