MA 122: Weekly HW 10

Answer these questions on a separate sheet of paper. Remember that your work must be very neat and complete.

Problem A:

(1) Use the definition of the Riemann integral of one variable to calculate
$$\int_{-1}^{1} 3x\,dx.$$

(2) Use the definition of the Riemann integral of two variables to calculate
$$\iint_R f\,dA$$
where $f(x,y) = xy$ and R is the rectangle in the xy plane with corners $(0,0)$, $(1,0)$, $(1,3)$, $(0,3)$. If you use lowerleft corners as sample points you will need the formula:
$$\sum_{k=1}^{n} (k-1) = \frac{(n-1)n}{2}.$$

(3) Let $f(x,y) = x^3$ and let R be the square in the xy plane with corners $(-1,-1)$, $(-1,1)$, $(1,-1)$, and $(1,1)$. Without doing any calculations, explain why
$$\iint_R f\,dA = 0.$$

(4) Suppose that $f\colon \mathbb{R}^3 \to \mathbb{R}$ is a continuous function. Let R be a cube in $\mathbb{R}^3$. Modelling your answer on the definition of the Riemann integral in one and two variables, create a definition of
$$\iiint_R f\,dV.$$
(Here V represents volume.)

Problem B:

(1) Let $f\colon \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = e^{x^2}$.
   (a) Find a series which represents f on the interval $[0,1]$. and

(b) Use your answer from (a) that to obtain a series which equals

$$\int_0^1 f\,dx.$$

(c) Use the first 10 terms of the series to estimate $\int_0^1 f\,dx$. You may wish to use Mathematica to do the computation.

(2) Let $f\colon \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = e^{x^2}$.

(a) Write $\int_0^1 f\,dx$ as the limit (as $n \to \infty$) of a Riemann sum.

(b) Some of the pieces of the Riemann sum in (a) depend on n. Write each of them as an expression in terms of n.

(c) Use $n = 10$ to estimate $\int_0^1 f\,dx$. You may wish to use Mathematica to help you with the computation.

(d) Use Mathematica to evaluate numerically $\int_0^1 f\,dx$. Which of the answers from (2.c) or (1.c) is more accurate?

(3) Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = \sqrt{1+xy}$. Let R be the rectangle with corners $(0,1)$, $(0,2)$, $(3,1)$, $(3,2)$.

(a) Use a subdivision of R into 4 subrectangles to estimate $\iint\limits_R f\,dA$. Be sure to specify the points $\mathbf{x}_{ij}^*$. You should probably use Mathematica to help with the computation.

(b) Use a subdivision of R into 16 subrectangles to estimate $\iint\limits_R f\,dA$. Be sure to specify the points $\mathbf{x}_{ij}^*$.

Problem C: In the 5th edition of the text, do problems 1, 3, 7, 9, 11, 14, 15, 16, 17, 23 on page 840. The answers to the odd numbered problems are in the back.

Problem D: Let $f(x,y) = -(x^2+y^2)+4$. The graph of f intersects the xy plane in a circle of radius 2 centered at the origin. Let S be the region between the xy plane and the graph of f.

(1) Set up an iterated integral in Cartesian coordinates representing the volume of S. Use Mathematica to solve the integral.

(2) Set up an iterated integral in polar coordinates representing the volume of S. Solve it without using Mathematica to find the volume of S.

Problem E: Find the mass of the pyramid with base in the plane $z = -6$ and sides formed by the three planes $y = 0$ and $y - x = 4$ and $2x + y + z = 4$ if the density of the solid is given by $\delta(x,y,z) = y$. (The mass of a solid P with density function δ is $\iiint\limits_P \delta\,dV$.

Problem F: In the 5th edition of the text, do problems 1, 3, and 24 on pages 850-851 and problems 19, 21, 22 on page 870.