**HW 3**

1. Let $(a_i)$ be a sequence such that each $a_i$ is one of the following numbers:

$$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

(a) Prove that the series

$$\sum_{i=1}^{\infty} \frac{a_i}{10^i}$$

converges. (Hint: Use the comparison test.)

(b) Explain the relevance of (a) to explaining why there exist numbers with an infinite number of digits to the right of the decimal point.

2. Consider the series

$$1 + 3x + 9x^2 + 27x^3 + 81x^4 + \dots$$

(a) Use the obvious pattern to write the series in summation notation.
(b) Explain why the series is a geometric series.
(c) For what values of $x$ does the series converge?

3. Determine whether or not the following series converge or diverge. If a sequence has both positive and negative terms also determine if the sequence is absolutely convergent.

(a) $\sum_{n=1}^{\infty} \frac{3}{n+2}$
(b) $\sum_{n=1}^{\infty} \frac{3}{\sqrt{3+n}}$
(c) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$
(d) $\sum_{n=1}^{\infty} \frac{1}{n^2+2n+2}$
(e) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$
(f) $\sum_{n=2}^{\infty} \frac{3^n}{(n-2)!}$
(g) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{3+n}}$
(h) $\sum_{i=0}^{\infty} \frac{(-1)^i \pi^{2i}}{(2i)!}$

4. Find the radius and interval of convergence for each of the following power series.

(a) $\sum_{i=0}^{\infty} \frac{x^{2i}}{(2i)!}$
(b) $\sum_{n=0}^{\infty} nx^n$
(c) $\sum_{n=0}^{\infty} (n!)x^n$
(d) $\sum_{n=0}^{\infty} 5^n x^n$

5. For each of the following functions, find its Maclaurin series and determine the radius and interval of convergence.

(a) $f(x) = \frac{1}{1-x}$.
(b) $f(x) = e^{x^2}$. (Hint: substitute $x^2$ into the Maclaurin series for $e^x$.)

6. Let $f(x) = |x - 2|$. Recall that this means that

$$f(x) = \begin{cases} -(x-2) & x \leq 2 \\ (x-2) & x \geq 2 \end{cases}$$

(a) Find a formula for the $n$th MacLaurin polynomial for $f(x)$.
(b) Find an upperbound (in terms of $x$) for the absolute value of the error $E_n(x)$ of the $n$th MacLaurin polynomial of $f(x)$ in terms of $x$ for $x \geq 0$.
(c) For what values of $x$ does the $|E_n(x)| \to 0$ as $n \to \infty$? What happens for the other values of $x$?
(d) What is the MacLaurin series for $f(x)$?
(e) What is the interval of convergence of the Maclaurin series for $f(x)$.
(f) Write a few sentences discussing the significance of the results of (c) and (e).