Calculus I Practice Exam 2

Instructions: The exam is closed book, closed notes, although you may use a note sheet as in the previous exam. A calculator is allowed, but you must show all of your work. **Your work is your answer.** If you have any questions, please ask immediately! Good luck.

(This practice exam is longer and more difficult than the actual exam.)

Problem #1 : Find the derivatives of the following functions (k is a constant).

a)

$$f(x) = \sqrt{1 + x\ln(7 + kx^2)}$$

b)

$$g(x) = (x^2 - e^{\sin(x^3 - x)})e^{k\cos(x)}$$

c)

$$h(x) = \frac{1 - \ln(2x)}{3 + e^{2x}}$$

Problem #2: Use the tangent line to the graph of $y = \tan(2x)$ at $x = \pi/6$ to approximate $\tan(2\pi/7)$.

Problem #3 :

a) Find dy/dx:

$$xy^2 = 2e^x - 2y$$

b) Find the equation of the line tangent to the above curve at the point $(0, 1)$.

Problem #4: Calculate the derivative of $y = \arccos(x)$. You must show all of the steps. You may find it helpful to think about triangles.

Problem #5:

Compute

$$\lim_{x\to 0} \frac{x}{\arctan(4x)}$$

Problem #6: A hanging rope can be modelled by the graph of the function:

$$f(x) = \frac{e^x + e^{-x}}{2}.$$

Find the global maxima and minima of $f(x)$ on the interval $[-2, 2]$.

Problem #7:

Use calculus to find all of the points where

$$f(x) = x^2 - |x - 1|$$

has local maxima and minima and to classify each as being a maximum or minimum.

Problem #8:

A lighthouse is located on an island 3 miles from a straight shoreline. There is a house on the shore directly opposite the lighthouse. The light on the lighthouse revolves at 4 revolutions per minute. How fast is the light travelling along the shore when it is one mile from the house?

Problem #9:

A trough, as shown in the diagram, contains eggnog half a foot deep. The organizer of the holiday party, adds eggnog to the trough at a rate of $0.5/h$ feet per minute, where h is the height of the eggnog. How fast is the eggnog rising when the height of the water is 2 feet? (Hint: The volume of a trapezoidal box (such as the one pictured) is $(b + B)hd/2$ where b is the length of the bottom, B is the length of the top, h is the vertical height of the trapezoid, and d is the depth of the box.)

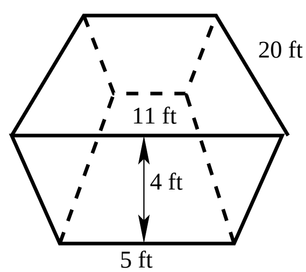

FIGURE 1. The eggnog trough.

Problem #10: Harpo Marx is on top of a 6 foot ladder leaning against a vertical wall. Zeppo Marx is pushing the base of the ladder towards the wall at .5 feet per second. How fast is Harpo rising when the angle between the base of the ladder and the floor is $60°$?

Problem #11: A producer of canned tuna is trying to figure out the optimal size of a tuna can. The can must hold 15 in^3 of tuna when initially packed. The can will be a cylinder of radius r and height h. The cost to make the sides of the can is \$.15/in^2. The cost to make the top of the can is \$.10/in^2. What is the least amount it will cost to make a can for tuna? (Hint: The volume of the cylinder is $V = \pi r^2 h$ and the surface area of the side of the can is $2\pi rh$ and the surface area of the top and bottom of the can (combined) is $2\pi r^2$.)

Problem #12:

Consider the function:

$$f(x) = 4 + x - 2x^3$$

on the interval $[0, 1]$.

a) Carefully explain why there is some number c in $[0, 1]$ such that $f(c) = 3.5$.

b) Carefully explain what the Mean Value Theorem says about this function on this interval.

Problem #13: Use the Mean Value Theorem to prove that if $f(x)$ is a function which is continuous on $[a, b]$ and differentiable on (a, b) such that $f'(x) = 0$ for all x in (a, b) then there is a constant k such that $f'(x) = k$ for all x in $[a, b]$.

Problem #14: Sketch part of a slope field for the differential equation:

$$y' = y - t$$

You slope field should include at least four slopes in each quadrant (and not on the axes).