PROBLEM LIST FROM AMS CENTRAL SPRING SECTIONAL MEETING, 2011

The following were compiled during a problem session at the Special Session on Thin Position at the Spring 2011 AMS Central Sectional Meeting at the University of Iowa. The session was organized by Jesse Johnson and Maggy Tomova. The name of the person suggesting each problem is listed in parentheses.

(1) (Yo'av Rieck) If K is a knot in bridge position with b bridges with respect to a genus g Heegaard surface H, we say that K has a (g,b) decomposition. We can form a $(g+1,b-1)$ decomposition of K by stabilizing H along one of the bridges of K. Using this stabilization procedure for a high distance knot K in S^3 we typically obtain the following sequence:

$$\begin{aligned}&(0,b) \to (1,b-1) \to (2,b-2) \to \dots \\ &\to (\text{genus}(E(K))-1, b-\text{genus}(E(K))+1) \to (\text{genus}(E(K)),0)\end{aligned}$$

where $\text{genus}(E(K))$ denotes the Heegaard genus of the exterior of K. Notice the large drop in bridge number at the last arrow. We should expect that there should be knots with a big drop in bridge number at any stage in this process. Construct such a knot. (Note that we can't use distance as a way to do this.)

(2) (Marty Scharlemann) Lickorish defined the rational bridge number $b_{\mathbb{Q}}$ for a knot K to be the minimal p/q where K has a (p,q) bridge position. Find an application of this invariant.

(3) (Yo'av Rieck) Negami's Conjecture: Let G be a finite connected graph having a finite cover $\widetilde{G}$ that embeds in S^2. Then G embeds in $\mathbb{R}P^2$.

It is known that if the graph $K_{2,2,2,1}$ does not have a cover that embeds in S^2, then the conjecture is true.

(4) (Marty Scharlemann) Inspired by Alex Zupan's examples of unknots that can't be thinned without increasing width: Invent a notion of thinness for unknotting discs which can be decreased to unknot the unknot.

(5) (Alexander Coward) Suppose that $S^1 \hookrightarrow \mathbb{R}^3$ is a knot with embedded normal bundle. Prove that there exists such an unknot that can't be isotoped to be the standard unknot without increasing the length or destroying the normal bundle. Try to discretize the problem?

(6) (Alexander Coward) In the limit, do most unknot diagrams monotonically decrease crossing number via Reidemeister moves?

(7) (Scott Taylor) Find a connection between quantum invariants or planar algebras and thin position or width.

(8) (Jesse Johnson) Define thin position for a surface in $\mathbb{R}^4$.

(9) (David Bachman) What is an explicit example of a knot in (global) thin position with compressible thin levels? (Zupan's example probably gives a locally thin example. Blair suggested that there is probably an example related to a counter example to additivity of width)

(10) (David Bachman) If a knot is in thin position, can you compress thin levels to get non-parallel essential surfaces?

(11) (Ryan Blair) Do there exist non-isotopic alternating links K_1 and K_2 having homeomorphic complements? If so, is it true that in each homeomorphism class there are only finitely many such alternating links?

(12) (Alex Zupan) If S is a surface that can be made almost normal in any given triangulation of M, is S incompressible or strongly irreducible? What about surfaces that can be made to have local indices summing to n in any given triangulation?

(13) (Alexander Coward) Suppose that $K \subset S^3$ is a smooth knot. Let $\mathscr{P}$ denote the set of all planes tangent to K at exactly 3 points. Is the trefoil knot the only knot with $\mathscr{P} = \varnothing$? On a related note, can we use topological techniques to understand geometric sweepouts?

(14) (Scott Taylor) Let C be a compact compressionbody and let $T \subset C$ be a spine. (That is, $\partial_+ C$ is isotopic to the frontier of $\partial_- C \cup T$.) Find an algorithm to construct all bridge surfaces for T in C. (Note that Taylor and Tomova have proved that such bridge surfaces are either trivial, stabilized, boundary-stabilized, perturbed, or T has a removable path.)