Assignment: 5
Due: Feb 14, 2024
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Exercise 2.5: Prove the following theorem:

Theorem 1. *Suppose that $n \in \mathbb{N}$. If n^2 is even then n is even.*

Before proving the theorem we recall the definition of even and odd.

Definition 2. *A number $n \in \mathbb{Z}$ is* ***even*** *if there exists $m \in \mathbb{Z}$ such that $n = 2m$. An integer $n \in \mathbb{Z}$ is* ***odd*** *if there exists $m \in \mathbb{Z}$ such that $n = 2m+1$.*

We will also need the following lemma. Its proof requires induction, so we omit it.

Lemma 3. *If $n \in \mathbb{Z}$ then n is even or n is odd, but not both.*

Proof of Theorem 1. We will prove the contrapositive. We assume that n is not even and we will show that n^2 is not even. Since n is not even, by Lemma 3, n is odd. By Definition 2 , there exists $m \in \mathbb{Z}$ such that $n = 2m + 1$. By algebra we have:

$$\begin{aligned} n^2 &= (2m+1)^2 \\ &= 4m^2 + 4m + 1 \\ &= 2(2m^2 + 2m) + 1. \end{aligned}$$

Since there exists $k = 2m^2 + 2m \in \mathbb{N}$ such that $n^2 = 2k + 1$, by Definition 2 n^2 is odd. Thus, by Lemma 3, n^2 is not even. Thus, we have shown that if n is not even then n^2 is not even and, by contraposition, that if n^2 is even then n is even. $\square$

Theorem 1 appears in [**?**].

Here are a few other examples of environments you might find useful:

- This
- is a bulleted list

(1) This is

(2) a numbered list

In the comments in the .tex file there is an example of how to include a figure.

References

[1] Last Name, First Name *A Math Book*, date, publisher.