Quiz Policies: This is a take home quiz. You are allowed to use the textbook and your notes, but are not allowed to discuss this quiz with anyone or use any other means of assistance. You may ask me for clarification of any of the questions. Remember, cheating is bad – don't do it. If you do cheat, you will get a zero on this quiz and, even worse, I will think ill of you. This is due at the *beginning* of class on the day indicated.

1. Suppose that a batch of 100 items contains 6 that are defective and 94 that are non-defective. If X is the number of defective items in a randomly drawn sample of 10 items from the batch, find:
    a. The probability that X = 0;
    b. The probability that X > 2.

$$A: P(X=0) = \frac{\binom{94}{10}}{\binom{100}{10}} = 0.522$$

$$\begin{aligned} B: P(X>2) &= 1 - P(X=0) + P(X=1) + P(X=2) \\ &= 1 - 0.533 - \frac{\binom{94}{9}\binom{6}{1}}{\binom{100}{10}} - \frac{\binom{94}{8}\binom{6}{2}}{\binom{100}{10}} \\ &= 0.01255 \end{aligned}$$

2. Consider a roulette wheel consisting of 38 numbers – 1 through 36, 0 and 00. If Smith always bets that the outcome will be one of the numbers 1 through 12, what is the probability that
    a. Smith will lose his first five bets;
    b. His first win will occur on his fourth bet;
    c. His first win will occur on his fifth bet, given that it does not occur in the first four.

$$A: P(\text{lose first 5}) = \left(\frac{26}{38}\right)^5 = 0.150$$

$$B: P(\text{first win on fourth bet}) = \left(\frac{26}{38}\right)^3 \frac{12}{38} = 0.101$$

$$\begin{aligned} C: P(Y=5 \mid Y>4) &= \frac{P(Y=5 \cap Y>4)}{P(Y>4)} = \frac{P(Y=5)}{P(Y>4)} \\ &= \frac{\left(\frac{26}{38}\right)^4 \frac{12}{38}}{\left(\frac{26}{38}\right)^4} = 0.316 \end{aligned}$$

3. Suppose the number of accidents occurring on a highway is a Poisson random variable with parameter $\lambda = 4$.
   a. Find the probability that 3 or more accidents occur today.
   b. Repeat part (a) under the condition that at least 1 accident occurs today.

$$A: P(X \ge 3) = 1 - P(X = 0) - P(X = 1) - P(X = 2)$$
$$= 1 - e^{-4} - 4e^{-4} - e^{-4}\frac{4^2}{2} = 0.762$$
$$B: P(X \ge 3 \mid X \ge 1) = \frac{P(X \ge 3)}{P(X \ge 1)} = \frac{0.762}{1 - e^{-4}} = 0.776$$

4. A box contains 5 red and 5 blue marbles. Two marbles are withdrawn randomly. If they are the same color, then you win \$1.10; if they are different colors, then you lose \$1.00. Calculate
   a. The expected value of the money you win;
   b. The variance of the amount you win.

Let X be the amount of winning,

$$P(X = 1.10) = \frac{4}{9} = 1 - P(X = -1.00)$$
$$E(X) = 1.1(4/9) + (-1.0)(5/9) = -0.067$$
$$Var(X) = [(1.1)^2(4/9) + (-1.0)^2(5/9)] - (-0.067)^2 = 1.09$$

5. Let X be a binomial random variable with parameters n and p. Show that $E\left[\frac{1}{X+1}\right] = \frac{1-(1-p)^{n+1}}{(n+1)p}$.
   Make sure you show all your work.

$$E[1/(X+1)] = \sum_{i=0}^{n} \frac{1}{i+1} \frac{n!}{(n-i)!i!} p^i (1-p)^{n-i}$$
$$= \sum_{i=0}^{n} \frac{n!}{(n-i)!(i+1)!} p^i (1-p)^{n-i}$$
$$= \frac{1}{(n+1)p} \sum_{i=0}^{n} \binom{n+1}{i+1} p^{i+1} (1-p)^{n-i}$$
$$= \frac{1}{(n+1)p} \sum_{j=1}^{n+1} \binom{n+1}{j} p^j (1-p)^{n+1-j}$$
$$= \frac{1}{(n+1)p} \left[1 - \binom{n+1}{0}^0 p^0 (1-p)^{n+1-0} \right]$$
$$= \frac{1}{(n+1)p} [1 - (1-p)^{n+1}]$$