Quiz Policies: This is a take home quiz. You are allowed to use the textbook and your notes, but are not allowed to discuss this quiz with anyone or use any other means of assistance. You may ask me for clarification of any of the questions. Remember, cheating is bad – don't do it. If you do cheat, you will get a zero on this quiz and, even worse, I will think ill of you. This is due at the *beginning* of class on the day indicated.

(1) A student has to sell 2 books from a collection of 6 math, 7 science, and 4 economics books. How many choices are possible if,

a. Both books are to be on the same subject;

$$\binom{6}{2}+\binom{7}{2}+\binom{4}{2}=42$$

b. The books are to be on different subjects?

There are (6)(7) choices of math and science, (6)(4) choices of math and economics, and (7)(4) choices of science and economics. 94 total choices.

(2) From a group of 8 women and 6 men a committee consisting of 3 men and 3 women is to be formed. How many different committees are possible if

a. 2 of the men refuse to serve together;

$$\binom{8}{3}\binom{4}{3}+\binom{8}{3}\binom{2}{1}\binom{4}{2}=896 \text{ possible committees}$$

There are $\binom{8}{3}\binom{4}{3}$ that do not contain either of the two men, and there are $\binom{8}{3}\binom{2}{1}\binom{4}{2}$ that contain exactly 1 of them.

b. 1 man and 1 woman refuse to serve together?

There are $\binom{7}{3}\binom{5}{3}+\binom{7}{2}\binom{5}{3}+\binom{7}{3}\binom{5}{2}=910$ possible committees consisting of neither fueding party, the feuding woman, and feuding man, serving, respectively.

(3) Find the simplest expression for the following events:

a. $(E \cap F) \cup (E \cap F^c)$

E

b. $(E \cap F) \cup (E^c \cap F) \cup (E \cap F^c)$

$E \cup F$

c. $(E \cap F) \cup (F \cap G)$
$(E \cup G) \cap F$

(4) Ninety-eight percent of all babies survive delivery. However, 15 percent of all births involve Cesarean (C) sections, and when a C section is performed the baby survives 96 percent of the time. If a randomly chosen pregnant woman does not have a C section, what is the probability that her baby survives?

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\begin{aligned}
&S:\{\text{survival}\}\\
&C:\{\text{delivery by C section}\}\\
&P(S \mid C) = 0.96; P(C) = 0.15\\
&P(S) = 0.98\\
&\qquad = P(S \mid C)P(C) + P(S \mid \bar{C})P(\bar{C})\\
&0.98 = (0.96)(0.15) + P(S \mid \bar{C})(0.85)\\
&P(S \mid \bar{C}) = 0.984
\end{aligned}
$$

(5) A recent college graduate is planning to take the first three actuarial exams in the coming summer. She will take the first actuarial exam in June. If she passes that exam, then she will take the second exam in July, and if she passes that one, then she will take the third exam in September. If she fails an exam, then she is not allowed to take any others. The probability that she passes the first exam is 0.9. If she passes the first exam, then the conditional probability that she passes the second exam is 0.8, and if she passes both the first and second exams, then the conditional probability that she passes the third exam is 0.7.

a. What is the probability she passes all three exams?

$$(0.9)(0.8)(0.7) = 0.504$$

b. Given that she did not pass all three exams, what is the conditional probability that she failed the second exam?

Let F_i denote the event she did not pass the i^{th} exam.

$$
\begin{aligned}
P(F_2 \mid (\bar{F}_1 \cap \bar{F}_2 \cap \bar{F}_3)^c) &= \frac{P(\bar{F}_1 \cap F_2)}{1 - P(\bar{F}_1 \cap \bar{F}_2 \cap \bar{F}_3)}\\
&= \frac{(0.9)(0.2)}{1 - 0.504} = 0.363
\end{aligned}
$$

(6) There are three relationships that hold if two events, A and B, are independent:
$P(A \mid B) = P(A)$;
$P(B \mid A) = P(B)$;
$P(A \cap B) = P(A)\, P(B)$
Show that if one of these equalities holds, then the other two must hold.