1a. In how many ways can 3 boys and 3 girls sit in a row?

The boys are girls are distinguishable (they're people after all!), so the order does matter.
{Bob, Joe, Bill, Mary, Jane, Liz} is NOT the same as {Joe, Bill, Bob, Mary,Jane, Liz} etc.
So we have $6! = 720$ ways.

1b. In how many ways can 3 boys and 3 girls sit in a row if the boys and girls are each to sit together?

There are 3! ways to order the boys, and 3! ways to order the girls.
We can also start the row with the boys, or the girls, so we have,
$2!3!3! = 72$

1c. In how many ways if only the boys must sit together?

If the boys must sit together, then we have 3! ways of ordering them. We also have 3! ways of ordering the girls. We also have 4 ways that we can place the 3 boys into the set of 3 girls - there can be 0 girls in the front, 1 girl in the front, 2 girls in the front, or all 3 girls in the front.
$4(3!)(3!) = 144$

1d. In how many ways if no two people of the same sex are allowed to sit together?
We can select any of the 6 to start the row, then we can select any of the 3 members of the opposite sex for the next spot. Then we have 2 to choose from for the third spot (of the same gender we began the row with), etc.
$(6)(3)(2)(2)(1)(1) = 72$

2. A person has 8 friends, 5 of whom will be invited to a party.
   a. How many choices are there if 2 of the friends are feuding and will not attend together?

There are two scenarios here - either neither of the feuding people comes, or one comes but not the other.

Total number of ways = number of ways when neither comes + number ways when one comes

$$\binom{6}{5}+\binom{2}{1}\binom{6}{4}$$

   b. How many choices if 2 of the friends will only attend together?

Again, we have two cases that we need to worry about. Either neither of the two friends come, or they both come.

Total number of ways = number of ways when both comes + number of ways when neither comes

$$\binom{6}{5}+\binom{6}{3}$$

3. Prove that $P(E \cup F^c) = P(E) - P(E \cup F)$

E can be written as $(E \cap F) \cup (E \cap \overline{F})$ so,

$P(E) = P[(E \cap F) \cup (E \cap \overline{F})]$

$(E \cap F)$ and $(E \cap \overline{F})$ are mutually exclusive events, so

$P(E) = P(E \cap F) + P(E \cap \overline{F})$

$P(E \cap \overline{F}) = P(E) - P(E \cap F)$

4. In a certain community, 36 percent of the families own a dog, and 22 percent of the families that own a dog also own a cat. In addition, 30 percent of the families own a cat. What is,

a) the probability that a randomly selected family owns both a dog and a cat?

$D:\{\text{own dog}\}$

$C:\{\text{own cat}\}$

$P(D) = 0.36$

$P(C) = 0.30$

$P(C \mid D) = 0.22$

$$\begin{aligned} P(C \cap D) &= P(C \mid D)P(D) \\ &= (0.22)(0.36) \\ &= 0.079 \end{aligned}$$

b) the conditional probability that a randomly selected family owns a dog, given it owns a cat?

$$\begin{aligned} P(D \mid C) &= \frac{P(C \mid D)P(D)}{P(C)} \\ &= \frac{(0.22)(0.36)}{0.30} \\ &= 0.264 \end{aligned}$$

5. At a psychiatric clinic the social workers are so busy that, on the average, only 60 percent of potential new patients that telephone are able to talk immediately with a social worker when they call. The other 40 percent are asked to leave their phone numbers. About 75 percent of the time a social worker is available to return the call on the same day, and the other 25 percent of the time the caller is contacted the following day. Experience at the clinic indicates that the probability a caller will actually visit the clinic for consultation is 0.8 if the caller was immediately able to speak to a social worker, whereas it is 0.6 and 0.4, respectively, if the patient's call was returned the same day or the following day.

a. What percentage of people that telephone visit the clinic for consultation?

I : {caller talks with worker immediately}

R : {caller talks with worker next day}

$\bar{R}$: {caller talks with worker second day}

V : {caller visits clinic}

$P(I) = 0.60$

$P(R \mid \bar{I}) = 0.75$

$P(\bar{R} \mid \bar{I}) = 0.25$

$P(V \mid I) = 0.80$

$P(V \mid (R \mid \bar{I})) = 0.60$

$P(V \mid (\bar{R} \mid \bar{I})) = 0.40$

$$\begin{aligned} P(V) &= P(V \mid I)P(I) + P(V \mid (R \mid \bar{I}))P(R \mid \bar{I})P(\bar{I}) + P(V \mid (\bar{R} \mid \bar{I}))P(\bar{R} \mid \bar{I})P(\bar{I}) \\ &= (0.8)(0.6) + (0.6)(0.75)(0.4) + (0.4)(0.25)(0.4) \\ &= 0.70 \end{aligned}$$

b. What percentage of patients that visit the clinic did not have their telephone calls returned?

They all had their calls returedn at some point! So 0%.