Mathematics 231

Lecture 9

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1

Announcements

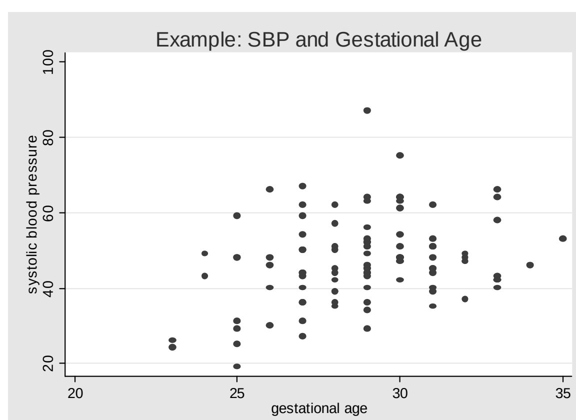
- Reading
  - Today M&M 2.3 117-119
    M&M 2.4 125-132
  - Next class M&M 2.5 142-151

2

Conditional Standard Deviation

- Conditional Standard Deviation
- Conditional SD in Regression
- Regression Assumptions
- Predicting Y from X versus Predicting X from Y

3



4

Conditional Mean

- The mean SBP of infants with a gestational age of 25 weeks in approximately 42 mm Hg.
- This is the **conditional** mean given a gestational age of 25 weeks, since it is based only on infants who satisfy some condition (25 weeks gestation).
- The **marginal** mean SBP is the mean SBP of all infants (47 mm Hg), regardless of gestational age.

Conditional Distributions – One more time

- In general, we can consider the distribution of Y variables (e.g., height) for observations that satisfy some condition $X = x$ (e.g., age equals 25 weeks).
- This is called the **conditional distribution of *Y* given *X = x*.**
- In a scatter plot, the conditional distribution of Y given $X = x$ is the distribution of points in the vertical strip above a given value of x.

Linear Regression

- Linear regression fits a straight line to the conditional mean of Y, given X.
- How might we determine the conditional SD at any given x=value?
- For example, what is the conditional SD of SBP for infants with a gestational age of 25 weeks?

Conditional Standard Deviation

- Conditional SD of Y given X = x:
  - In a scatterplot, the conditional standard deviation of Y given X = x is the spread of points in the vertical strip above a given value of x.
- The spread is determined relative to the center (mean) of the distribution of points in the vertical strip.

Conditional Standard Deviation

- **Conditional SD of Y given X = x:**
- The spread can be determined by the residuals:

$$y_i - \hat{y}_i = y_i - (a + bx_i)$$

- In calculating the SD, should we consider the spread of points only in the vertical strip above the particular value of x (e.g., 25 weeks)?

Recall: Regression Assumptions

- The regression line estimates the conditional mean of Y given $X=x$ for any point x if the following assumptions are met.
  1. Conditional mean of Y is a linear function of X.
  2. Conditional SD of Y is constant for all X.
- We often make an additional assumption:
  3. The conditional distribution of Y is a normal distribution for any value of x.

Conditional Standard Deviation

- **Conditional SD of Y given X = x:**

$$s_{y|x} = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}$$

- Measures the degree of scatter of the points about the regression line (in any give vertical strip).
- In Stata, this is denoted by **Root Mean Square Error (MSE).**
- This is the variation NOT explained by the linear regression model.

Example: Height and Age

TABLE 2.7 Mean height of Kalama children

Age x in months	Height y in centimeters
18	76.1
19	77.0
20	78.1
21	78.2
22	78.8
23	79.7
24	79.9
25	81.1
26	81.2
27	81.8
28	82.8
29	83.5

Example: Height and Age

```
. regress height age

      Source |       SS       df       MS              Number of obs =      12
-------------+------------------------------           F(  1,    10) =  880.00
       Model |  57.6548678     1  57.6548678           Prob > F      =  0.0000
    Residual |  .655171562    10  .065517156           R-squared     =  0.9888
-------------+------------------------------           Adj R-squared =  0.9876
       Total |  58.3100394    11  5.30091267           Root MSE      =  .25596

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      height |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |   .6349653   .0214047    29.66   0.000     .5872726     .682658
       _cons |   64.92832    .508409   127.71   0.000     63.79551    66.06112
------------------------------------------------------------------------------
```
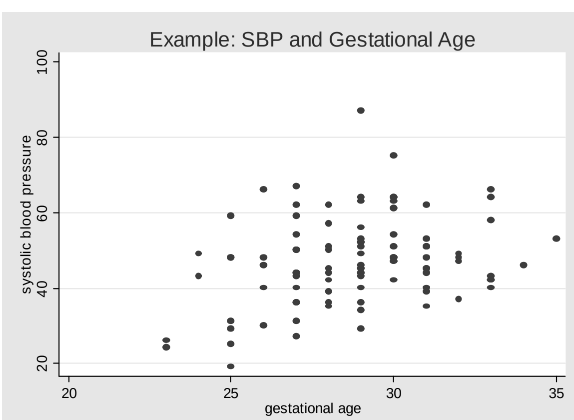

13

Example: SBP and Gestational Age

- Data: SBP and gestational age for 100 infants.
- Mean Gestational age = 28.9 weeks, SD = 2.53 weeks.
- Mean SBP = 47.1 mm Hg, SD = 11.4 mm Hg
- Correlation between gestational age and SBP, r=0.28.
- Suppose we are interested in predicting SBP from gestational age.

14



15

Example: SBP and Gestational Age

- Regression line:

    SBP = 10.6 + 1.26 (gestational age)

    Conditional SD = 11

Of infants 25 weeks in gestation, what proportion have a SBP between 31 and 53 mm Hg.

16

Example: SBP and Gestational Age

- If we make assumption (3), then the SBP of 25-week old infants have a normal distribution with mean = 10.6 + 1.26 (gestational age).
- What's the SD of this conditional distribution? 11 mm Hg.
- Of 25-week old infants, what proportion have an SBP between 31 and 53 mm Hg?

17

Recall: The Empirical Rule

- All normal distributions have the following property:
- 68% of the area under the curve lies with σ of the mean.
- 95% of the area of the curve lies within 2σ of the mean.
- 99.7% of the area of the curve lies within 3σ of the mean.

18

Example: SBP and Gestational Age

- For 25-week old infants, SBP's between 31 and 53 mm Hg are 1 SD above and below the mean (42 mm Hg for 25-week old infants).
- So, 68% of 25-week old infants have SBP's between 31 and 53 mm Hg.
- **Previously:** Calculating the proportion of **all** infants with SBP's between 31 and 53.
- **Now:** Can calculate for infants of a given age only.

19

Predicting X from Y

- Regression line:

  SBP = 10.6 + 1.26 (gestational age)
- For an infant 25 weeks into gestation, our prediction for its SBP is

  SBP = 10.6 + 1.26 (25) = 42 mm Hg
- Now consider an infant with an SBP of 42 mm Hg, what is our prediction of its gestational age?

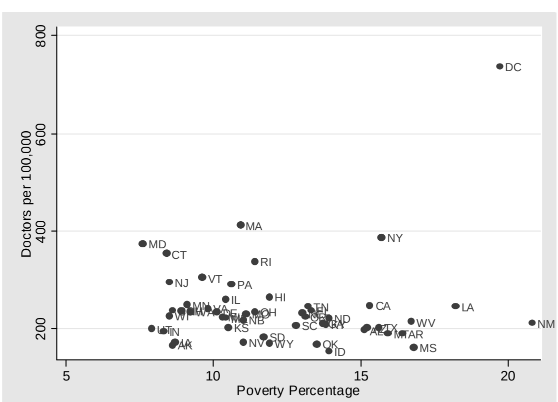
20

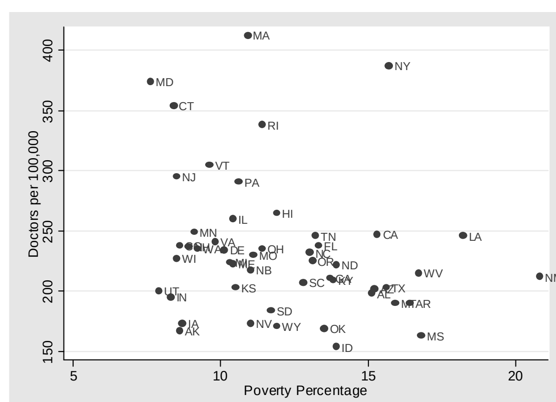
Predicting X from Y

- Consider the scatterplot of SBP (Y) versus gestational age (X).
- Then the conditional mean gestation age of infants with an SBP of 42 mm Hg is the mean of the points within the horizontal strip at Y = 42.
- In general, predicting Y from X is NOT the same as predicting X from Y (although the data in this example provide similar regression results).

Example: Poverty and Doctors

- Between 1997 and 1999, data were collected on poverty rates and the number of doctors in each of the 50 states and DC.
- Of interest is how strongly poverty and the number of doctors in related.
- How do we expect these two to relate?





Example: Poverty and Doctors

```
. regress doctors poverty if state != "DC"

      Source |       SS       df       MS              Number of obs =      50
-------------+------------------------------           F(  1,    48) =    2.70
       Model |  8670.18752     1  8670.18752           Prob > F      =  0.1067
    Residual |  154011.032    48  3208.56318           R-squared     =  0.0533
-------------+------------------------------           Adj R-squared =  0.0336
       Total |   162681.22    49   3320.0249           Root MSE      =  56.644

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     doctors |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     poverty |  -4.375241   2.661601    -1.64   0.107    -9.726749    .9762674
       _cons |   287.0354   33.04209     8.69   0.000     220.5998     353.471
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```
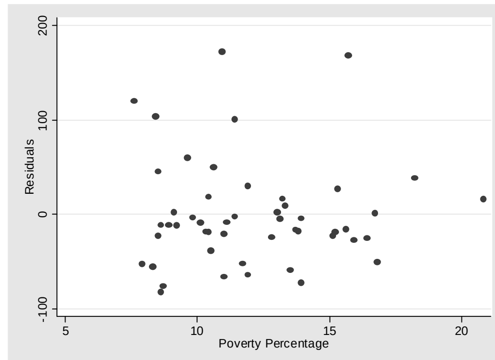
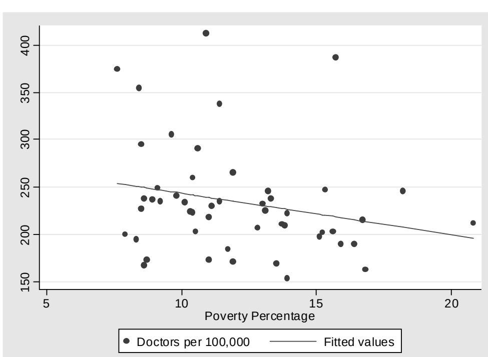


Residual Plot



Example: Poverty and Doctors



Reverse the Prediction

```
. regress poverty doctors if state!="DC"

      Source |       SS       df       MS              Number of obs =      50
-------------+------------------------------           F(  1,    48) =    2.70
       Model |  24.1387972     1  24.1387972           Prob > F      =  0.1067
    Residual |  428.784391    48  8.93300815           R-squared     =  0.0533
-------------+------------------------------           Adj R-squared =  0.0336
       Total |  452.923188    49  9.24333037           Root MSE      =  2.9888

------------------------------------------------------------------------------
     poverty |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     doctors |  -.0121812   .0074102    -1.64   0.107    -.0270804     .002718
       _cons |   14.89854   1.787209     8.34   0.000     11.30511    18.49196
------------------------------------------------------------------------------
```

- Is this the same as the line we'd get by rearranging terms from the previous regression? NO