Mathematics 231

Lecture 8

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Announcements

- Reading
  - Today — M&M 2.3 — 117-119
    M&M 2.4 — 125-132
  - Next class — M&M 2.5 — 142-151

Evaluating the Regression Model

- Checking Assumptions
- Residual Plots
- R^2 and Correlation

Assumptions

- The regression line estimates the conditional mean of Y given $X=x$ for any point x if the following assumptions are met.
  1. Conditional mean of Y is a linear function of X.
  2. Conditional SD of Y is constant for all X.
- We often make an additional assumption:
  3. The conditional distribution of Y is a normal distribution for any value of x.

Checking Assumptions

- **Model and Residuals**

  Data = Predicted Values + Residuals

  (Pattern) (Deviation)

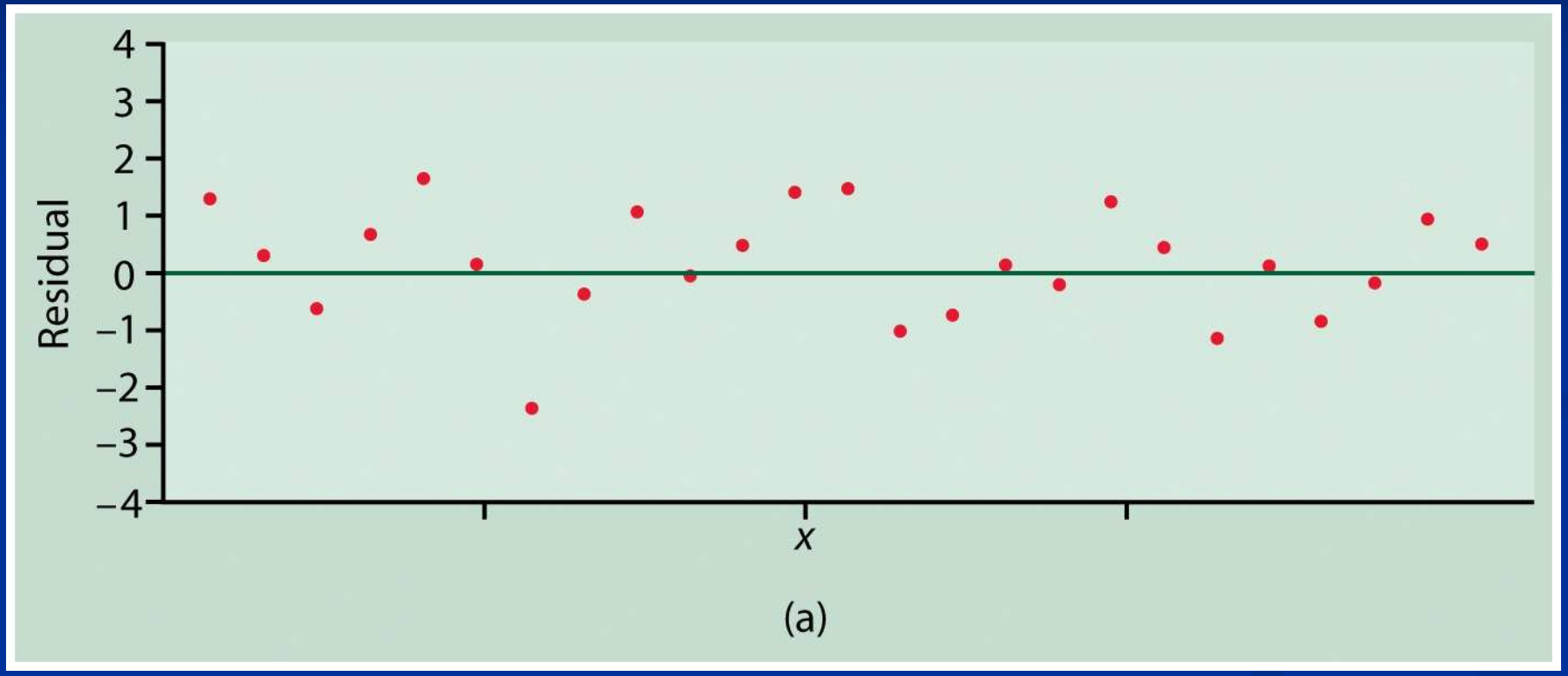
- **Predicted values:** the part of the data that is explained by the regression model.
- **Residuals:** the part of the data that is not explained by the regression model.

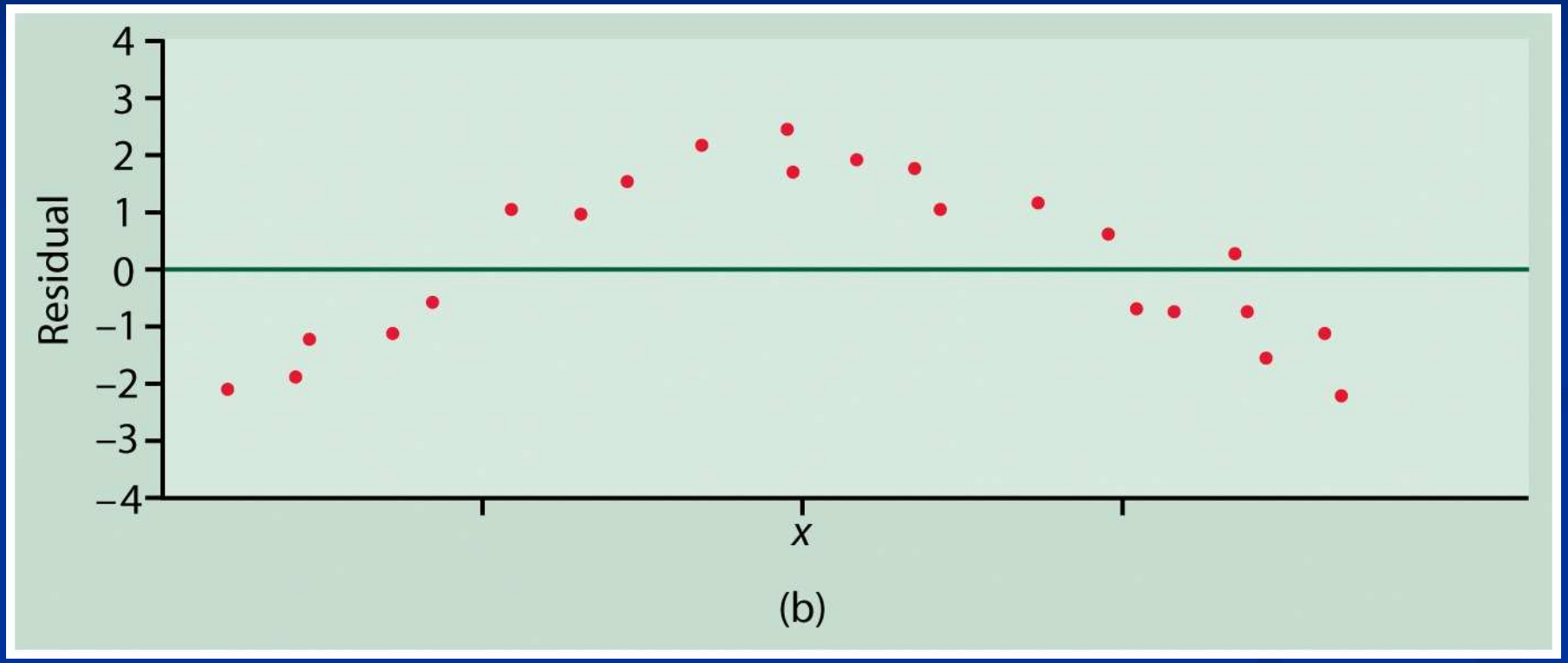
Checking Assumptions

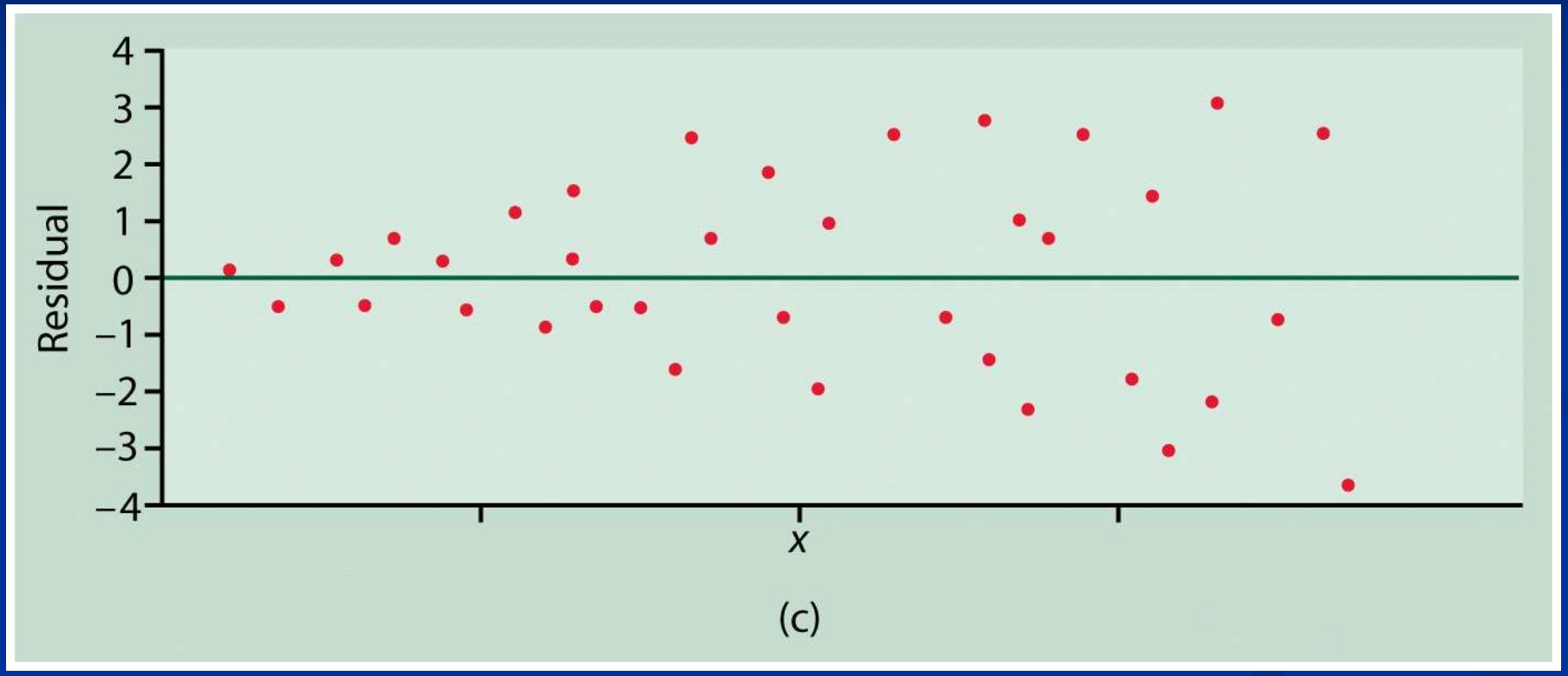
- In linear regression,
  - Fitted line represents the pattern.
  - Residuals represent deviations from the pattern.
- Examine the scatter plot of the original data with the regression line superimposed on it.
- Do you see any marked deviations from the line?

Residual Plots

- You always need to use a **residual plot** to check to see if model assumptions hold. Plot the residuals against the x variable (or predicted values).
- What to look for in a residual plot:
  - There should be no obvious patterns (random scatter about 0).
  - Vertical spread of the points should be approximately the same over the entire range of x-values.


Residual
4
3
2
1
0
−1
−2
−3
−4
x
(a)


Residual
4
3
2
1
0
−1
−2
−3
−4
x
(b)


Residual
4
3
2
1
0
−1
−2
−3
−4
x
(c)

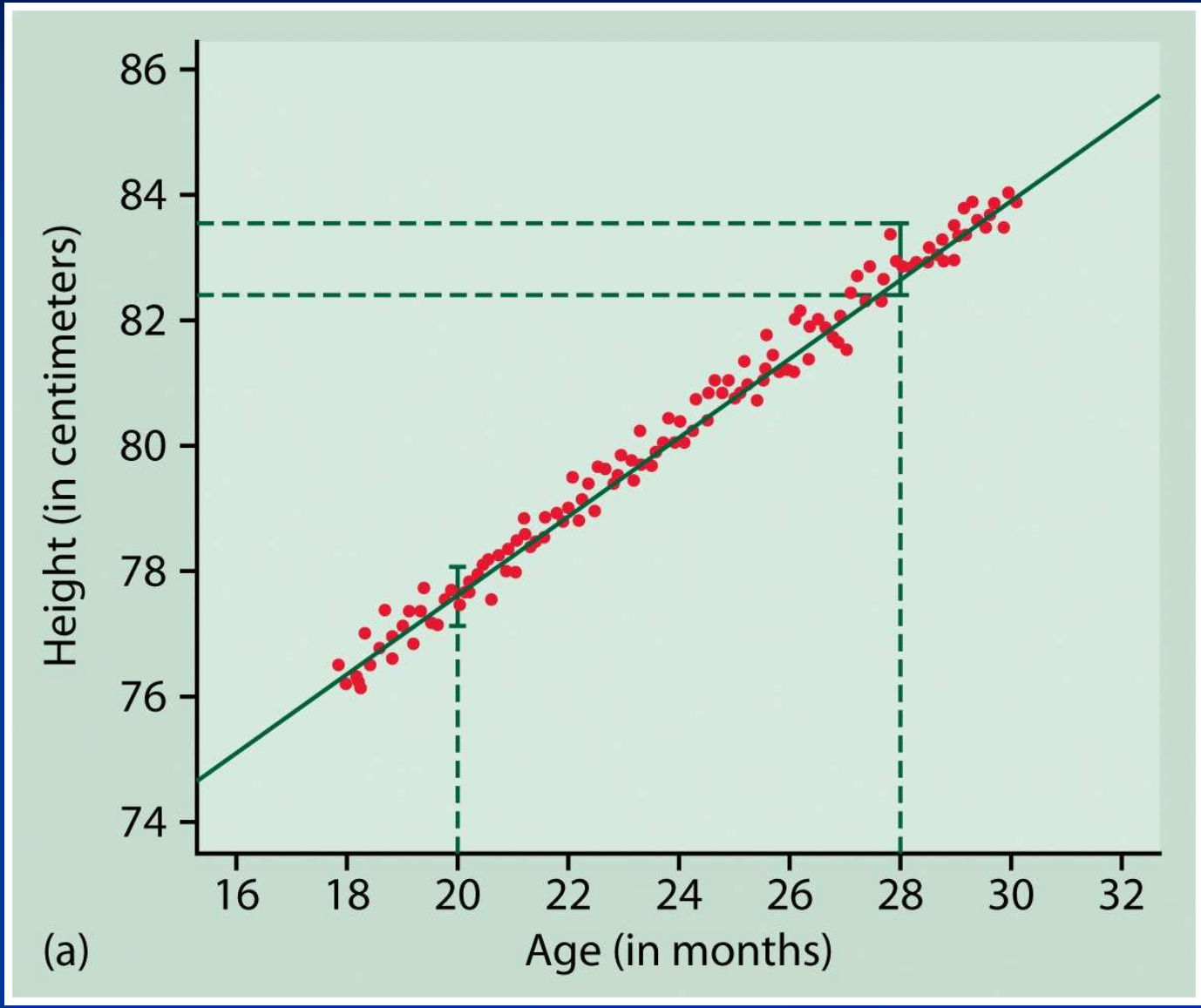
Residual Plots in Stata

- After you run the regression, click on **Statistics > Linear regression and related > Regression diagnostics > Residual versus predictor plot**
- Enter the name of the explanatory variable in the box.

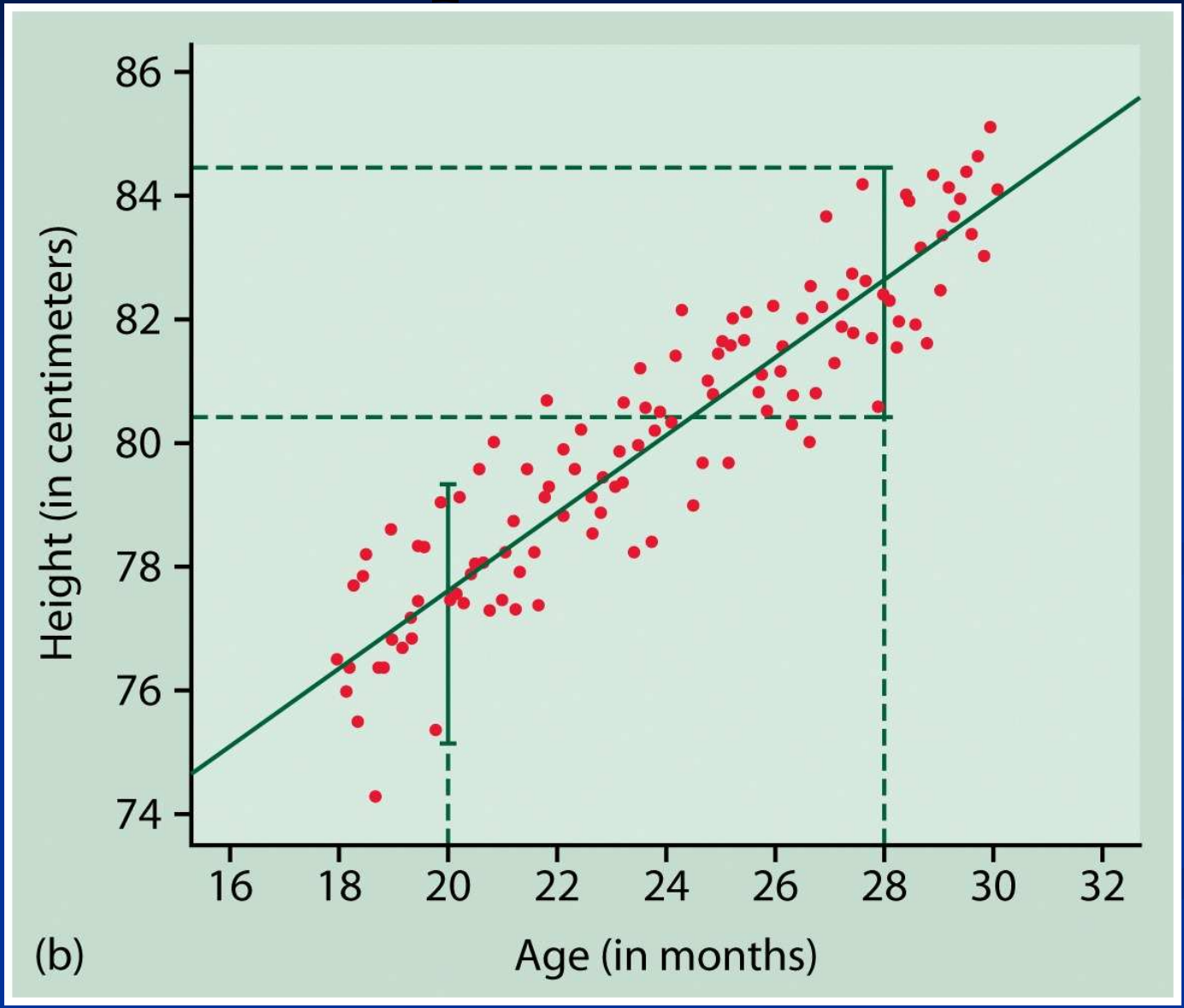
R^2: Measure of Fit

- R^2 = squared value of the correlation coefficient
- Since the correlation must be between -1 and 1, R^2 must be between 0 and 1.
- R^2 has the interpretation of being the proportion of variation in *Y* that is explained by the variation in *X*.

Example: $R^2 = 0.989$



Example: $R^2 = 0.849$



R^2: Measure of Fit

- If $R^2 = 0.80$, that does not mean that 80% of Y is explained by X.

- This is a common mistake that is made.

Pizza Example

- Small pizza costs $5.90 plus $1 per topping.
- 100% of the variation in small pizza prices is due to differences (variation) in the number of toppings.
- Not the same as saying that 100% of the price of small pizzas is due to toppings.