Mathematics 231

Lecture 7

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1

Announcements

- Reading
  - Today M&M 2.3 108-121
  - Next class M&M 2.3 117-119
    M&M 2.4 125-132

2

Linear Relationships & Regression

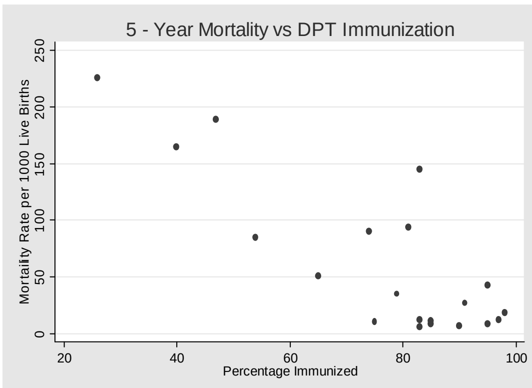
- Linear relationship between two variables.
- Response and explanatory variables.
- Regression line.
- Least squares criterion.

3

Response and Explanatory Variables

- A **response variable**, denoted as Y, measures the outcome of an experiment, survey, or study. Y is the variable we want to explain or predict.
- An **explanatory variable**, denoted as X, is a variable that may affect, explain or predict (but not necessarily cause) the response variable.

4



5

Example: Height and Age

TABLE 2.7 Mean height of Kalama children	
Age x in months	Height y in centimeters
18	76.1
19	77.0
20	78.1
21	78.2
22	78.8
23	79.7
24	79.9
25	81.1
26	81.2
27	81.8
28	82.8
29	83.5

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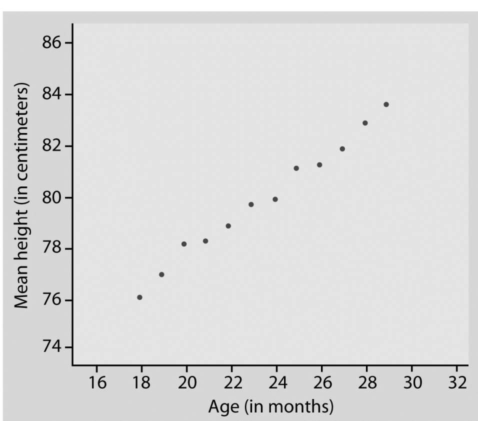

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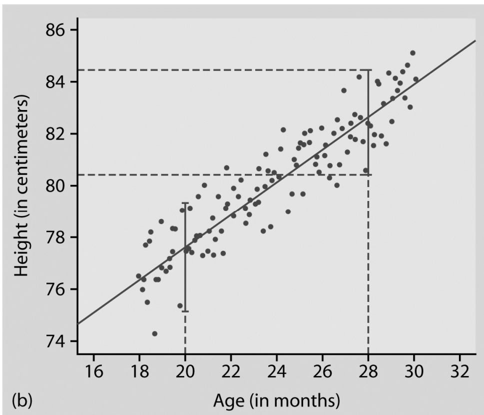
Conditional Distributions

- In general, we can consider the distribution of Y variables (e.g., height) for observations that satisfy some condition $X = x$ (e.g., age equals 28 months).
- This is called the **conditional distribution of *Y* given *X = x*.**
- In a scatter plot, the conditional distribution of Y given $X = x$ is the distribution of points in the vertical strip above a given value of x.

8



Conditional Mean

- Conditional distributions have center, spread, and shape properties like all distributions.
- The mean value of Y in the vertical strip above a given value x is called the **conditional mean of Y given $X = x$**.

Linear Regression

- Linear regression is used to explain or predict Y using X.
- It quantifies the relationship between the two variables in terms of a straight line.
- Suppose we have n pairs of Y and X,

$$(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots, (x_n y_n)$$

- How can we find the straight line that best "fits" or describes these data?

Linear Regression

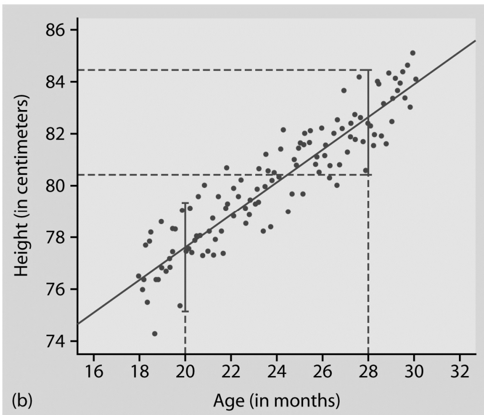
- This line has an equation of the form:

$$\hat{y}_i = a + bx_i$$

where $\hat{y}_i$ (y-hat) is the predicted value of Y,

a is the y-intercept (the value of Y when $X = 0$),

and b is the slope of the line.

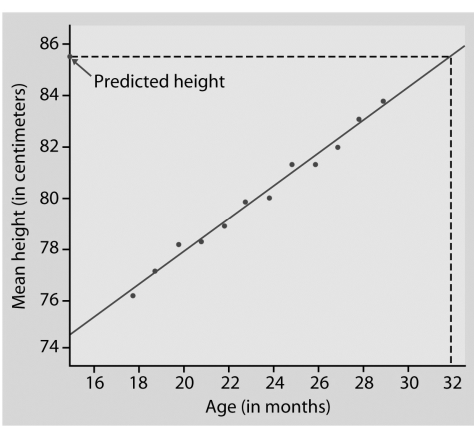


Definition 1

- For any particular value, x_i, the **predicted (or fitted)** value is:

$$\hat{y}_i = a + bx_i$$

and is the y-value of the line at x_i.

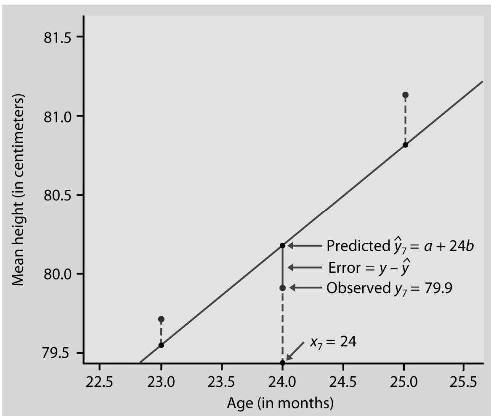


Definition 2

- The vertical deviation from a point to the line (the difference between the observed and predicted values of Y, or the error) is called the **residual**.

$$y_i - \hat{y}_i = y_i - (a + bx_i)$$

$$\text{residual} = \varepsilon_i = \text{observed } y_i - \text{ predicted } y_i$$



17

Least Squares Criterion

- The "best fit" line is defined as the line that minimizes the sum of the squared residuals.
- We want the values of *a* and *b* that minimizes the following quantity,

$$\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = \sum_{i=1}^{n}(y_i - (a + bx_i))^2$$

18

Least Squares Intercept and Slope

- The values of *a* and *b* that minimize this quantity are,

$$b = r\frac{s_y}{s_x}$$

$$a = \overline{y} - b\overline{x}$$

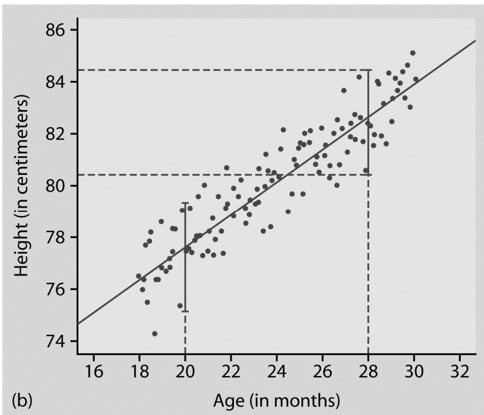
where r is the correaltion coefficient.

19

Assumptions

- The regression line estimates the conditional mean of *Y* given *X*=*x* for any point *x* if the following assumptions are met.
  1. Conditional mean of *Y* is a linear function of *X*.
  2. Conditional SD of *Y* is constant for all *X*.
- We often make an additional assumption:
  3. The conditional distribution of *Y* is a normal distribution for any value of *x*.

20



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Example: Height and Age

```
. regress height age

      Source |       SS       df       MS              Number of obs =      12
-------------+------------------------------           F(  1,    10) =  880.00
       Model |  57.6548678     1  57.6548678           Prob > F      =  0.0000
    Residual |  .655171562    10  .065517156           R-squared     =  0.9888
-------------+------------------------------           Adj R-squared =  0.9876
       Total |  58.3100394    11  5.30091267           Root MSE      =  .25596

------------------------------------------------------------------------------
      height |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |   .6349653   .0214047    29.66   0.000     .5872726     .682658
       _cons |   64.92832    .508409   127.71   0.000     63.79551    66.06112
------------------------------------------------------------------------------
```