Mathematics 231

Lecture 5

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Announcements

- Reading
  - Today M&M 1.3 62-71
  - Next class M&M 2.1 83-94
    M&M 2.2 101-104

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Calculating Normal Probabilities

- Normal Distribution
- Calculating Normal Probabilities
- The 68-95-99.7 Rule (a.k.a. the "empirical rule")

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Probability Distributions

- Every random variable has a corresponding probability distribution.
- This distribution allow us to determine the probabilities associated with specified ranges of values.
- The probabilities represent the **relative frequency** of occurrence **in a large number of trials under essentially identical conditions**.

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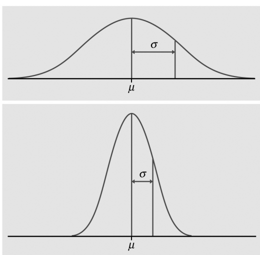
Probability Distributions

- These can be displayed graphically, or with mathematical formulae:



$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(x-\mu)^2}{2\sigma} \right]$$

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Probability Density Curves

- The probability of a quantitative variable is represented by a probability density curve.
- The probability associated with a specified range of values is determined by the area beneath the curve that lies between the two values.
- "Probability" = "Area"

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Review: Normal Distribution

- All normal distributions have the same shape: bell-shaped curve.
- They differ in their mean, μ, and SD, σ.
- The curve is symmetric about its mean, μ.
- Total area under the curve is 1.
- Special Case: The **standard normal** distribution has mean = 0, and SD = 1.

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The Empirical Rule

- All normal distributions have the following property:
- 68% of the area under the curve lies with σ of the mean.
- 95% of the area of the curve lies within 2σ of the mean.
- 99.7% of the area of the curve lies within 3σ of the mean.

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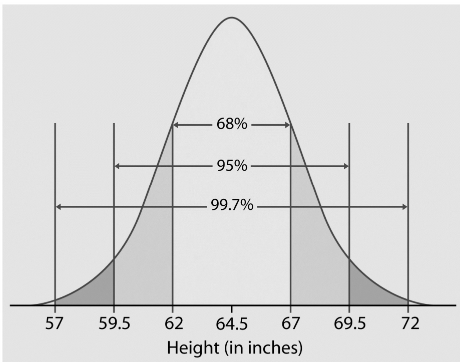
Example: Heights of Women

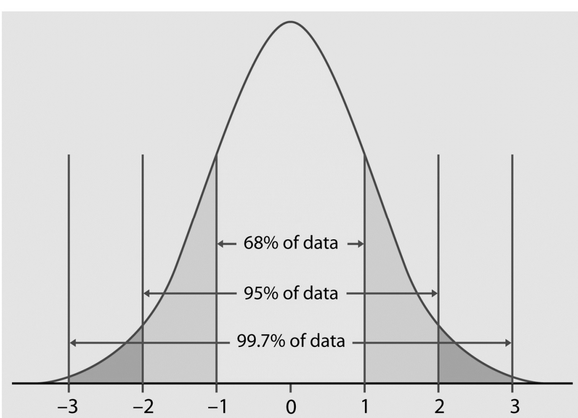


Standardizing and Z-scores

- All normal distributions are identical if measured in units of σ about their mean, μ.
- If x has a normal distribution with mean μ and SD σ (often denoted $X \sim N(\mu,\sigma)$), then

$$z = \frac{x-\mu}{\sigma}$$

has a standard normal distribution, or $Z \sim N(0,1)$.



Example: Dear Abby

- Dear Abby: You wrote in your column that a woman is pregnant for 266 days. Who said so? I carried my baby for 10 months and 5 days, and there is no doubt about it because I know the exact date my baby was conceived. My husband was in the Navy and it couldn't have possibly been conceived any other time…I don't drink or run around, and there is no way the baby isn't his, so please print a retraction about the 266-day carrying time because I am in a lot of trouble.

  \- San Diego Reader

Normal Calculations

- When presented with a problem that involves calculations of normal probabilities:
  1. Always draw a picture!
  2. Standardize
- Recall: Once we have standardized, we only need a single table of probabilities for N(0,1).
- What table? We'll get to it in just a little bit.

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Example: Dear Abby

- According to well-documented norms, the distribution of gestation time is approximately normal with mean 266 days and SD of 16 days.
- What percent of babies have a gestation time greater than or equal to 310 days (10 months and 5 days)?
- How do we figure this out? Draw a picture and standardize.

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Example

- If x has a normal distribution, μ = 266 days and σ = 16 days, we have:

$$P(x \geq 310) = P\left(\frac{x-266}{16} \geq \frac{310-266}{16}\right)$$

$$P(z \geq 2.75) = P(z \leq -2.75) = 0.003$$

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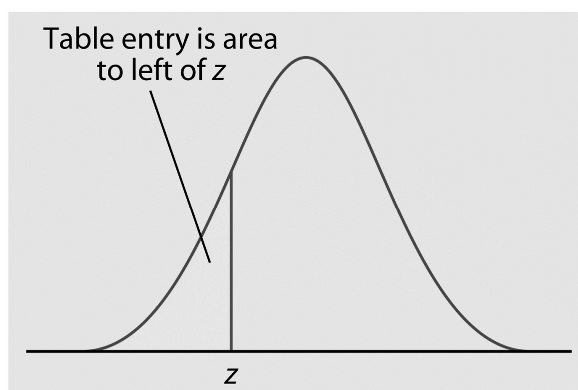

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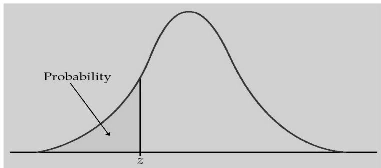

Table entry for z is the area under the standard normal curve to the left of z.

TABLE A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379

TABLE A Standard normal probabilities

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−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011
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−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244
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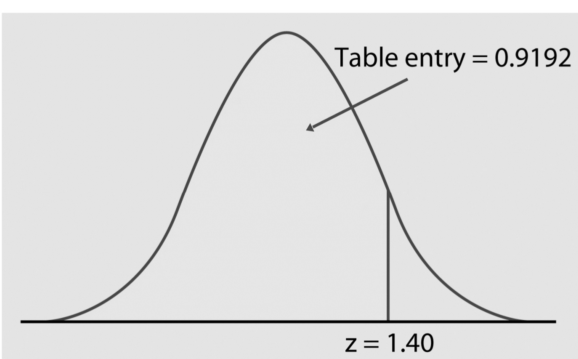

Example: NCAA Reqs

- NCAA requires athletes to score at least 820 on combined math and verbal parts of SAT exam in order to compete in their first year.
- In 2000, combined SAT scores had an approximately normal distribution with mean = 1019, and SD = 209.
- What percentage of all students have scores > 820?

Example: NCAA Reqs

- If x has a normal distribution, $\mu = 1019$ and $\sigma = 209$ days, we have:

$$P(x \le 820) = P\left(\frac{X - 1019}{209} \le \frac{820 - 1019}{209}\right)$$

$$P(z \le -0.95) = 0.17$$

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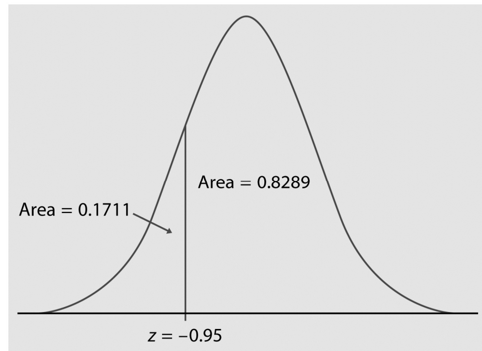
Example: NCAA Reqs



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Reverse Normal Calculations

- Sometimes need to determine the value of z with a particular area to the left (or right) of z
- For example, what z has an area of 0.1 to the right?
- Since Table A only gives area to the left, we need to state the problem as:
- What z has an area of 0.9 to the left?

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Example: SAT Verbal Scores

- If x has a normal distribution with mean = 505, and SD = 110, what x will place student in the top 10%?
- Probability that $x \ge ? = 0.1$ is the same as asking probability that $z \le (? - 505)/110 = 0.1$, where z has a standard normal distribution.

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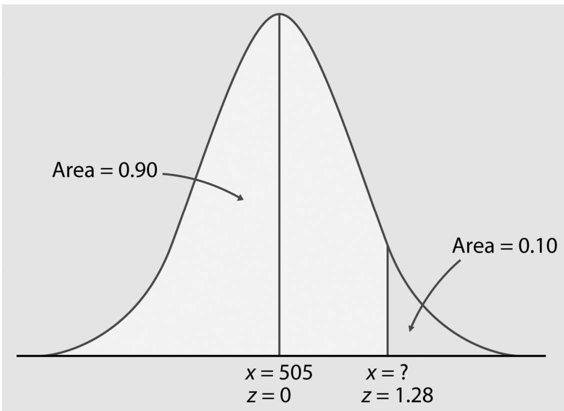

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Example: SAT Verbal Scores

- Probability $z \leq 1.28 = 0.9$ (from Table A)
- Probability $z \geq 1.28 = 0.1$
- To determine the SAT score, set
  $(x - 505)/110 = 1.28$
  and solve the equation for x.

$x = 505 + (1.28)(110) = 645.8$

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