Mathematics 231

Lecture 32

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Announcements

- Today
  - Multiple Regression
  - Effect Modification

Modifying Variables

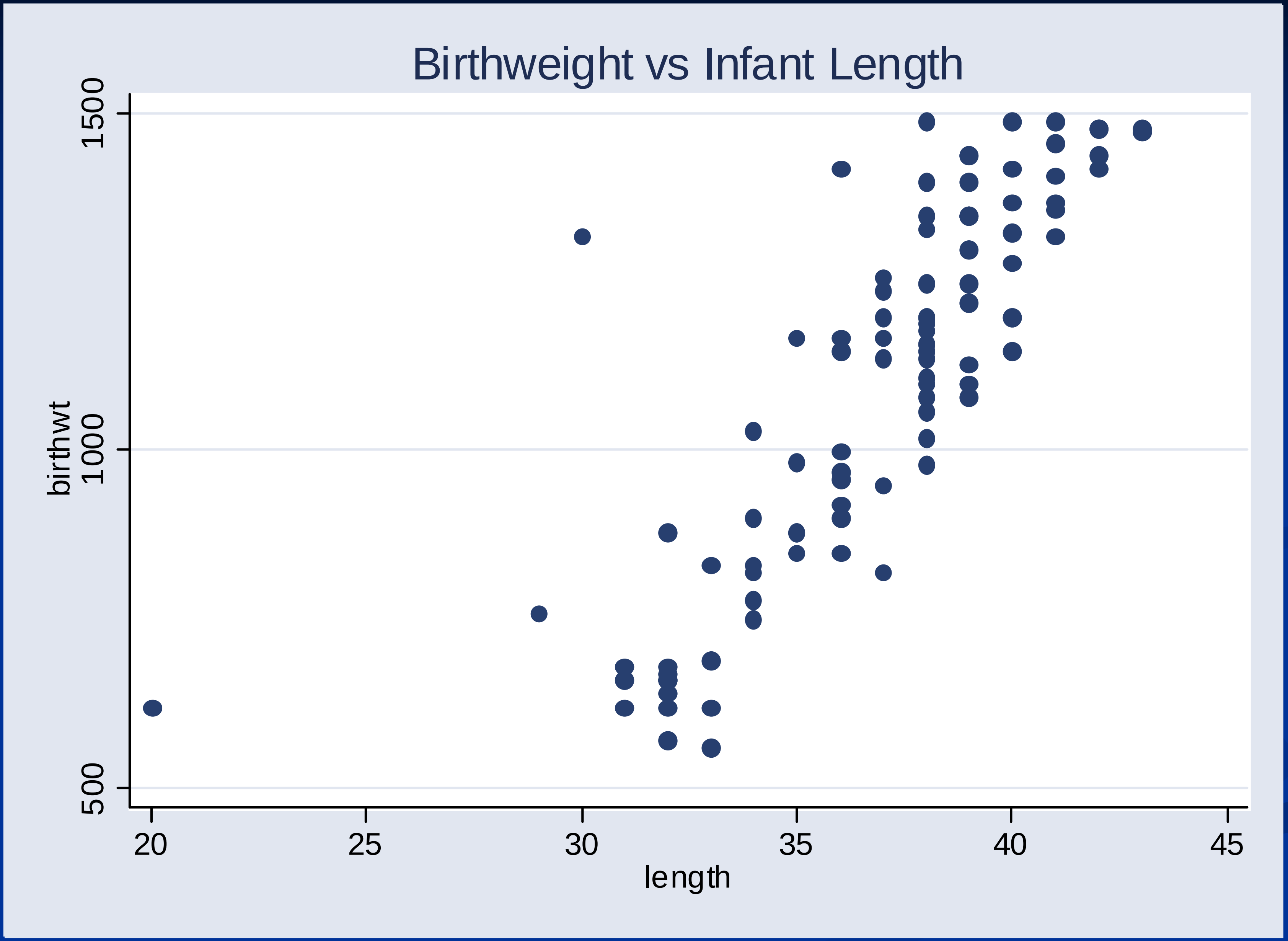
- A **modifying variable** changes the effect of a predictor on the outcome.
- Modifying variables are included in the model by adding an **interaction term**.
- These are generated by multiplying two covariates (predictors) together.
- When an interaction term is present, you generally want to include its **main effects** regardless of the p-values associated with them.

Modifying Variables

- For example, if you have two predictors, x1 and x2, the interaction term would be x1*x2.
- If x1*x2 is a statistically significant predictor, then x1 and x2 must also remain individually in the model.
- The effect of x1 on the outcome is dependent upon the value of x2.
- Interactions can include more than two main effects, but their interpretations become difficult.

Example: Birthweights

- Let's consider the birthweight data.
- We already know that toxemia, by itself, is not a significant predictor, but that with gestational age it is.
- Now consider infant length as a predictor of birthweight.


Birthweight vs Infant Length
birthwt
1500
1000
500
20
25
30
35
40
45
length

Example: Birthweights

```
. gen lentox=length*toxemia

. regress birthwt length toxemia lentox

      Source |       SS       df       MS              Number of obs =     100
-------------+------------------------------           F(  3,    96) =   72.08
       Model |  4997922.76     3  1665974.25           Prob > F      =  0.0000
    Residual |  2218819.99    96  23112.7083           R-squared     =  0.6925
-------------+------------------------------           Adj R-squared =  0.6829
       Total |  7216742.75    99  72896.3914           Root MSE      =  152.03

------------------------------------------------------------------------------
     birthwt |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      length |   57.47765   4.690349    12.25   0.000     48.16738    66.78792
     toxemia |  -1192.974   443.1596    -2.69   0.008    -2072.639   -313.3094
      lentox |   30.50423    11.7999     2.59   0.011     7.081609    53.92686
       _cons |  -1007.631   172.6113    -5.84   0.000    -1350.262   -665.0001
------------------------------------------------------------------------------
```

Example: Birthweights

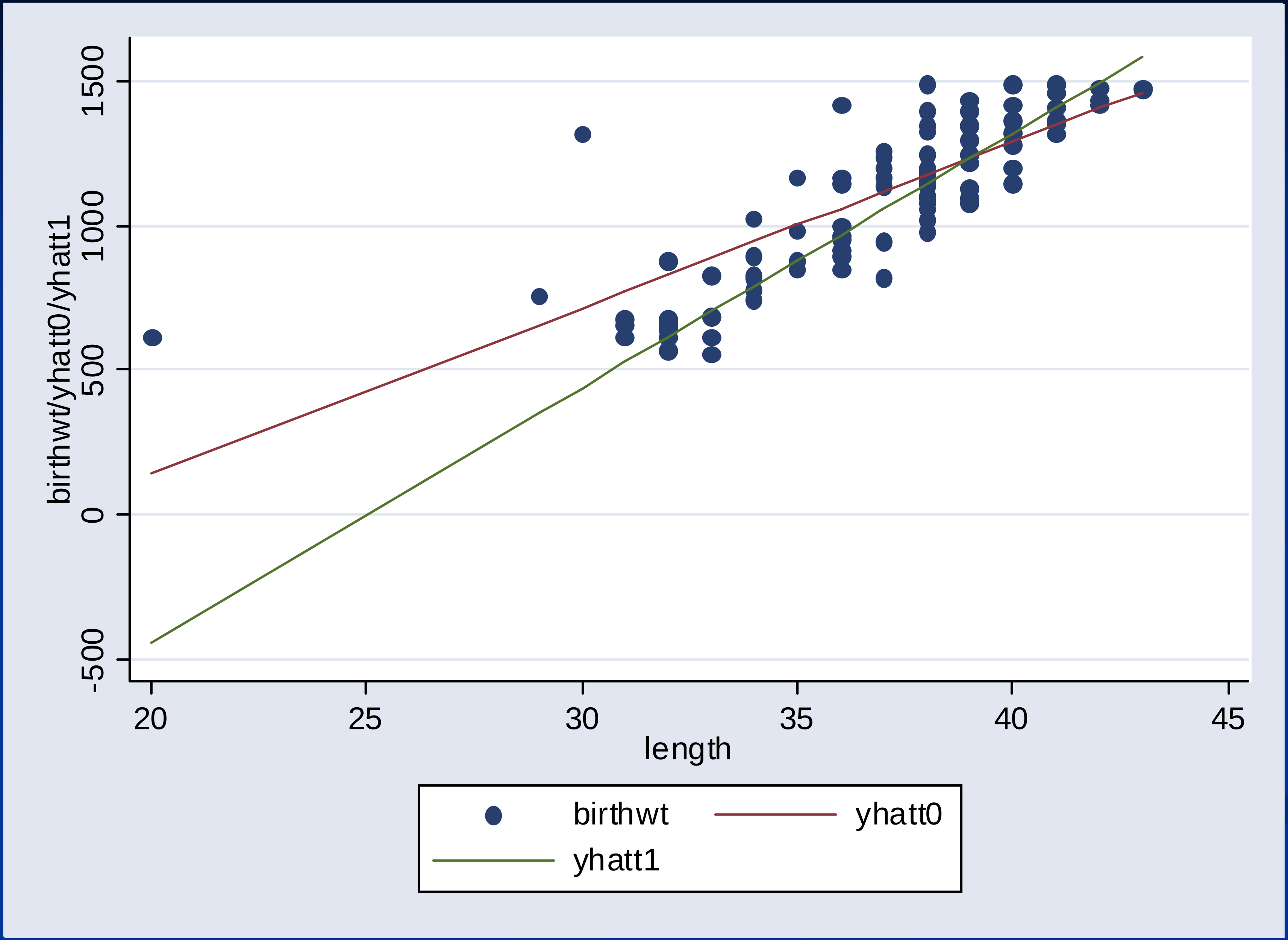
- The effect of infant length on birthweight depends on whether toxemia is present or not.
- E(Y)= -1008+57.5(length)-1193(toxemia)+30.5(length*toxemia)
- For those with toxemia:

  E(Y)= -2201+88(length)
- For those without toxemia:

  E(Y)= -1008+57.5(length)

Plotting Both Regression Lines

- Knowing the two regression equations (from setting toxemia to 0 and 1), use Stata to generate two sets of predicted values.
- . gen yhatt0=-1008+57.5*length
- . gen yhatt1=-2201+88*length
- Then use the "Overlaid twoway graphs" to place these on the same plot as the observed data.



birthwt/yhatt0/yhatt1
1500
1000
500
0
-500
20
25
30
35
40
45
length
birthwt
yhatt0
yhatt1

The Effect of a Modifying Variable

- The interaction term has changed the slope.
- The interaction term is of primary interest when its coefficient is significantly different from zero.
- The terms that make up the interaction must be singly included in the model (the main effects).
- Often, these main effects may not individually have coefficients that are significantly different from zero.
- They also often do not have meaningful interpretations if an interaction term is present.

Collinearity

- Collinearity is always present to some degree when you include an interaction term.
- **Colliearity** occurs when two or more explanatory variables are correlated to the extent that they convey essentially the same information about the variation in the response.
- One symptom of colliearity is the instability of the estimated coefficients and their standard errors (i.e., the standard errors become large).

Okay, So I'm Confused

- This is the point… there is no set-in-stone method for finding the most appropriate model.
- The goal is to find a model that balances prediction (e.g., a large R^2) with parsimony (not too many predictors).
- There are several semi-standard methods for doing this, and may result in different final models.

Choosing Predictors

- Ideally, we should have some prior knowledge as to which variables might be relevant.
- To study fully all of the predictors, it would be necessary to run a separate regression analysis for each possible combination of variables.
- While such a procedure would be thorough, it would be terribly time consuming.
- More frequently we use a stepwise approach to choose the best-fitting model.

Stepwise Regression

- Most statistical packages have automated routines built in that will perform a systematic method for obtaining the best model.
- The simplest of these are the **forward selection** and **backward selection** methods.
- The automated routines are not recommended for use by any self-respecting statistician, but they can be done "manually." We won't do them here.
- The best model results from careful statistical thought, combined with subject matter considerations.

Forward Selection

- **Forward selection** begins with nothing in the model and introduces one variable at a time.
- The variable that has the smallest p-value for its coefficient is added (or largest test statistic).
- This variable is kept in the model, the from the remaining explanatory variables, we repeat the procedure.
- We do this until no variables, when introduced to the model, have an adequately small p-value (or cause the estimates of the other coefficients to become unstable).

Backward Selection

- **Backward selection** begins with all the variables in the model and drops the one with the largest p-value (smallest test statistic).
- From the remaining variables the one with the next largest p-value is dropped.
- We repeat this procedure until all of the remaining variables have small p-values.
- Note that it is entirely possible that the final models will differ depending on whether forward, or backward, selection is used.

Stepwise Regression

- More complicated procedures exist in which variables may be added after they have been dropped (or dropped after they've been added), according to certain rules.
- These procedures are called **stepwise regression** procedures, and are generally preferred.
- However, they are difficult to do without using an automated computer routine.
- What about interaction terms?