Mathematics 231

Lecture 31

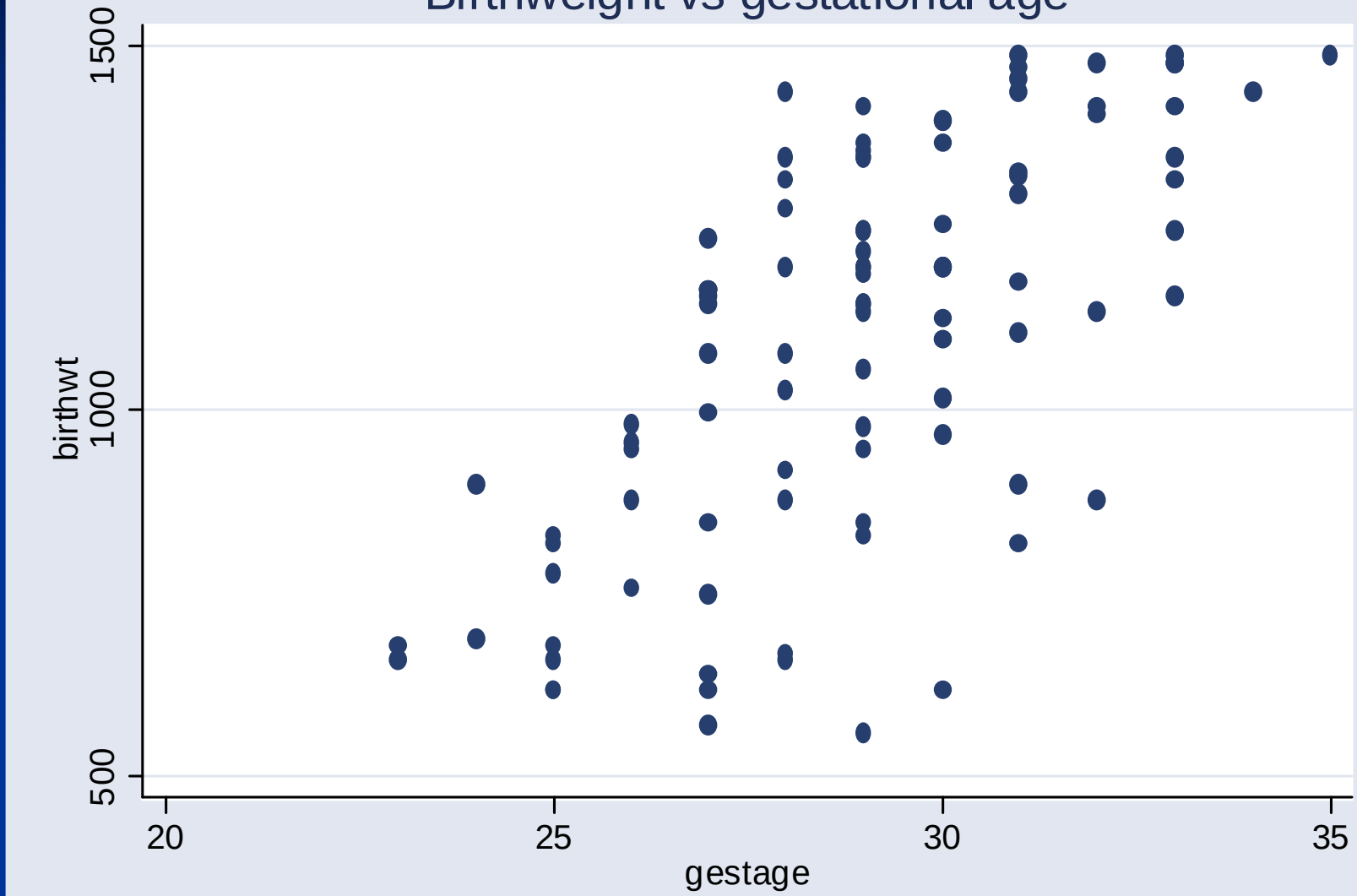
Liam O'Brien

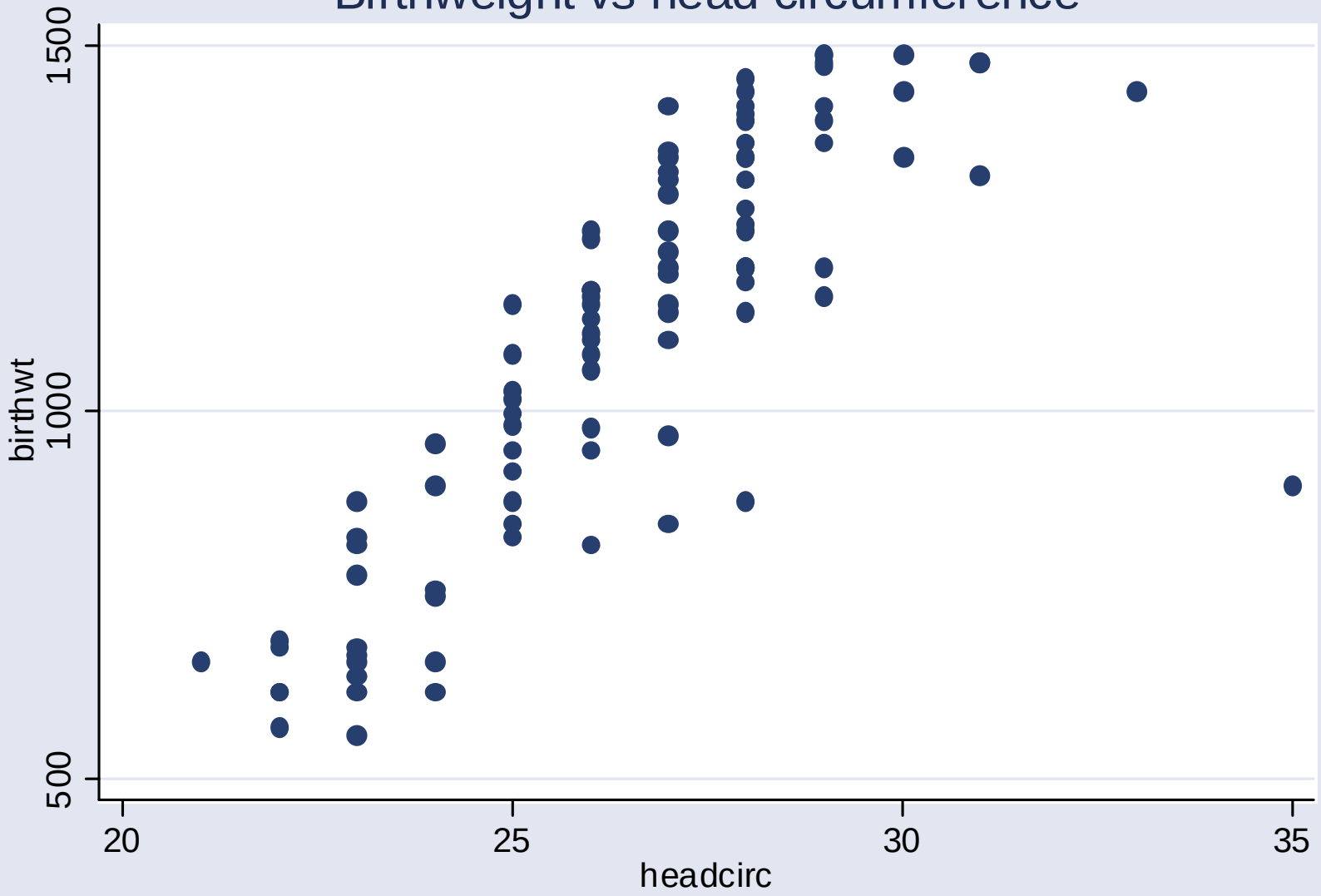
Announcements

- Reading
  - Today M&M 11

Example: Birthweights

- Consider the birthweight data that we used in class before.
- We predicted birthweight with infant length before, but now also consider two new variables.
- Data was also obtained on the presence of toxemia, gestational age, head circumference, and mother's age.
- We next examine the predictive power of head circumference and gestational age on birthweight.


Birthweight vs gestational age
birthwt
1500
1000
500
20
25
30
35
gestage


Birthweight vs head circumference
birthwt
500
1000
1500
20
25
30
35
headcirc

Comparison of Predictors

- We see a positive relationship between both predictors and the response.
- This is common and we often look at the correlations among all variables (response and predictors).
- Highly correlated predictors often have overlapping predictive information about the response.

Example: Birthweights

```
. pwcorr birthwt gestage headcirc

             |  birthwt  gestage headcirc
-------------+---------------------------
     birthwt |   1.0000
     gestage |   0.6599   1.0000
    headcirc |   0.7988   0.7807   1.0000
```

Example: Birthweights

```
. regress birthwt gestage

      Source |       SS       df       MS              Number of obs =     100
-------------+------------------------------           F(  1,    98) =   75.61
       Model |  3143019.07     1  3143019.07           Prob > F      =  0.0000
    Residual |  4073723.68    98   41568.609           R-squared     =  0.4355
-------------+------------------------------           Adj R-squared =  0.4298
       Total |  7216742.75    99  72896.3914           Root MSE      =  203.88

------------------------------------------------------------------------------
     birthwt |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     gestage |   70.30993   8.085854     8.70   0.000     54.26382    86.35604
       _cons |  -932.4039   234.4884    -3.98   0.000    -1397.738   -467.0693
------------------------------------------------------------------------------
```

Example: Birthweights

```
. regress birthwt headcirc

      Source |       SS       df       MS              Number of obs =     100
-------------+------------------------------           F(  1,    98) =  172.82
       Model |  4605298.87     1  4605298.87           Prob > F      =  0.0000
    Residual |  2611443.88    98  26647.3866           R-squared     =  0.6381
-------------+------------------------------           Adj R-squared =  0.6344
       Total |  7216742.75    99  72896.3914           Root MSE      =  163.24

------------------------------------------------------------------------------
     birthwt |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    headcirc |   85.17802   6.479268    13.15   0.000     72.32013    98.03592
       _cons |  -1154.109   172.1523    -6.70   0.000    -1495.739    -812.478
------------------------------------------------------------------------------
```

Example: Birthweights

```
. regress birthwt gestage headcirc

      Source |       SS       df       MS              Number of obs =     100
-------------+------------------------------           F(  2,    97) =   86.79
       Model |  4629638.61     2  2314819.31           Prob > F      =  0.0000
    Residual |  2587104.14    97  26671.1767           R-squared     =  0.6415
-------------+------------------------------           Adj R-squared =  0.6341
       Total |  7216742.75    99  72896.3914           Root MSE      =  163.31

------------------------------------------------------------------------------
     birthwt |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     gestage |    9.90101   10.36436     0.96   0.342    -10.66938     30.4714
    headcirc |   77.44205   10.37285     7.47   0.000     56.85482    98.02929
       _cons |  -1235.533    192.166    -6.43   0.000    -1616.929   -854.1362
------------------------------------------------------------------------------
```

Example: Birthweights

- The usefulness of gestational age as a predictor of birthweight has now become statistically insignificant (at the 0.05 level).
- This is happening because both gestational age and head circumference provide overlapping information on birthweights.
- Notice what happened to R-squared – it increased from 0.63 to 0.64 with the addition of head circumference to the model.
- What about the interpretation of the coefficients?

R^2 and Adjusted R^2

- The R^2 value will always increase with the addition of a new predictor.
- Having too many predictors is BAD, so we penalize the R^2 value based on the number of predictors in the model – this is the adjusted R^2.
- Note that adjusted R^2 decreased slightly.
- This indicates that adding head circumference did not help us predict birthweight much.
- Note that the adjusted R^2 does not have the same interpretation as R^2.

R^2 and Adjusted R^2

- Use of the adjusted R^2 in evaluating regression model is highly controversial.
- It is safe to say, however, that too many predictors (i.e., the kitchen sink model) is NOT a good thing.
- We only have a finite amount of information and want to put as much as possible to work in estimating the error.
- However, we lose a degree of freedom for each predictor.

Examining Correlations

```
. pwcorr birthwt gestage length headcirc momage

             |  birthwt  gestage   length headcirc   momage
-------------+------------------------------------------------
     birthwt |   1.0000
     gestage |   0.6599   1.0000
      length |   0.8156   0.6752   1.0000
    headcirc |   0.7988   0.7807   0.7127   1.0000
      momage |   0.1546   0.2658   0.2180   0.1321   1.0000
```

Example: Birthweights

```
. regress birthwt toxemia

      Source |       SS       df       MS              Number of obs =     100
-------------+------------------------------           F(  1,    98) =    0.01
       Model |  1005.40823     1  1005.40823           Prob > F      =  0.9072
    Residual |  7215737.34    98  73629.9729           R-squared     =  0.0001
-------------+------------------------------           Adj R-squared = -0.0101
       Total |  7216742.75    99  72896.3914           Root MSE      =  271.35

------------------------------------------------------------------------------
     birthwt |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     toxemia |    7.78481   66.61992     0.12   0.907    -124.4203    139.9899
       _cons |   1097.215   30.52908    35.94   0.000     1036.631    1157.799
------------------------------------------------------------------------------
```

Example: Birthweights

```
. regress birthwt gestage toxemia

      Source |       SS       df       MS              Number of obs =     100
-------------+------------------------------           F(  2,    97) =   51.91
       Model |  3730907.03     2  1865453.51            Prob > F      =  0.0000
    Residual |  3485835.72    97  35936.4507            R-squared     =  0.5170
-------------+------------------------------           Adj R-squared =  0.5070
       Total |  7216742.75    99  72896.3914            Root MSE      =  189.57

------------------------------------------------------------------------------
     birthwt |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     gestage |   84.05796   8.250832    10.19   0.000     67.68234    100.4336
     toxemia |  -206.5908   51.07771    -4.04   0.000     -307.966   -105.2157
       _cons |    -1286.2   234.9178    -5.48   0.000    -1752.447   -819.9535
------------------------------------------------------------------------------
```

Example

- The regression equation is given by:

  E(birthwt)=-1286+84.1(gestage)-206.6(toxemia)
- The variable "toxemia" is an indicator variable (or dummy variable) that indicates presence of toxemia.
- For those with toxemia:

  E(birthwt)=-1492+84.1(gestage)
- For those without toxemia:

  E(birthwt)=-1286+84.1(gestage)

Interpreting Coefficients

- Note that the interpretation of any one coefficient is conditional on holding all the other coefficients constant.
- In the case of a binary predictor, we can create two "separate" regression lines based on setting the indicator to 0 or 1.

Birthweight vs Gestage