Mathematics 231

Lecture 23

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Announcements

- Reading
  - Today — M&M 7.1 — 422-428
  - Next class — M&M 6.3 — 494-399
    M&M 6.4 — 401-410

Topics

- Hypothesis testing
  - One-sample t-test

Hypothesis Testing and Confidence Intervals

Consider a 95% confidence interval for μ.

Values of μ falling outside the interval are not compatible with the data and can be rejected at the $\alpha = 0.05$ significance level.

In general, a level α (two-sided) significance test rejects $H_0 : \mu = \mu_0$ when the value of H_0 falls outside a level (1-α)% confidence interval for μ.

Example: Birthweights

A sample of 50 children born to mother's who smoke in Norway gives mean weight of $\bar{x}$=3200g.

A 95% confidence interval for μ is given by,

$$\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}} = 3200 \pm 1.96\frac{430}{\sqrt{50}} = (3080, 3319)$$

Because $\mu_0 = 3500$ is not in this 95% CI, we can reject the null hypothesis $H_0 : \mu = 3500$ at the α=0.05 level.

Hypothesis Tests and Confidence Intervals

If $H_0 : \mu = \mu_0$ is rejected at 0.05 significance level then a 95% CI for μ would not contain μ_0.

If $H_0 : \mu = \mu_0$ is not rejected at 0.05 significance level then a 95% CI for μ would contain μ_0.

Hypothesis test results in a p-value, but only allows us to make a statement about the single value μ_0.

Confidence interval provides a range of plausible values for μ, but does not give a p-value.

Test for a Population Mean μ (σ unknown)

Given an SRS of size, n, we want to test $H_0 : \mu = \mu_0$ against $H_A : \mu \neq \mu_0$ (two-sided).

Use test statistic: $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

When $H_0 : \mu = \mu_0$ is true, this statistic has a t-distribution with n-1 degrees of freedom.

Example: Vitamin C

- U.S. Agency for International Development purchased 238,300 metric tons of corn soy blend.
- The CSB is fortified and used in emergency relief in countries throughout the world.
- It is critical that vitamin levels be within a certain range in order to be safe and effective.
- For vitamin C this level is 40mg/100g.
- A sample of size 8 produced a mean vitamin C concentration of 22.50 with a sample SD of 7.2.

Example: Vitamin C

$H_0 : \mu = 40;\ H_A : \mu \neq 40$

A sample of 8 gave a mean concentration of

$\bar{x}$=22.5, s=7.2

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

P-value < 0.05; therefore we reject H_0 at the α=0.05 level and conclude that the mean vitamin C concentration is not 40, and is significantly lower than 40.