

Mathematics 231

Lecture 23
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Announcements

- Reading
 - Today M&M 7.1 422-428
 - Next class M&M 6.3 494-399
 - M&M 6.4 401-410

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Topics

- Hypothesis testing
 - One-sample t-test

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Hypothesis Testing and Confidence Intervals

Consider a 95% confidence interval for μ .

Values of μ falling outside the interval are not compatible with the data and can be rejected at the $\alpha = 0.05$ significance level.

In general, a level α (two-sided) significance test rejects $H_0 : \mu = \mu_0$ when the value of H_0 falls outside a level $(1-\alpha)\%$ confidence interval for μ .

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Example: Birthweights

A sample of 50 children born to mother's who smoke in Norway gives mean weight of $\bar{x}=3200$ g.

A 95% confidence interval for μ is given by,

$$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}} = 3200 \pm 1.96 \frac{430}{\sqrt{50}} = (3080, 3319)$$

Because $\mu_0 = 3500$ is not in this 95% CI, we can reject the null hypothesis $H_0 : \mu = 3500$ at the $\alpha=0.05$ level.

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Hypothesis Tests and Confidence Intervals

If $H_0 : \mu = \mu_0$ is rejected at 0.05 significance level then a 95% CI for μ would not contain μ_0 .

If $H_0 : \mu = \mu_0$ is not rejected at 0.05 significance level then a 95% CI for μ would contain μ_0 .

Hypothesis test results in a p-value, but only allows us to make a statement about the single value μ_0 .

Confidence interval provides a range of plausible values for μ , but does not give a p-value.

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Test for a Population Mean μ (σ unknown)

Given an SRS of size, n , we want to test $H_0 : \mu = \mu_0$ against $H_A : \mu \neq \mu_0$ (two-sided).

Use test statistic:
$$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

When $H_0 : \mu = \mu_0$ is true, this statistic has a t -distribution with $n-1$ degrees of freedom.

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Example: Vitamin C

- U.S. Agency for International Development purchased 238,300 metric tons of corn soy blend.
- The CSB is fortified and used in emergency relief in countries throughout the world.
- It is critical that vitamin levels be within a certain range in order to be safe and effective.
- For vitamin C this level is 40mg/100g.
- A sample of size 8 produced a mean vitamin C concentration of 22.50 with a sample SD of 7.2.

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Example: Vitamin C

$$H_0 : \mu = 40; H_A : \mu \neq 40$$

A sample of 8 gave a mean concentration of

$$\bar{x}=22.5, s=7.2$$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

P-value < 0.05; therefore we reject H_0 at the $\alpha=0.05$ level and conclude that the mean vitamin C concentration is not 40, and is significantly lower than 40.