Mathematics 231

Lecture 19

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Announcements

- Reading
  - Today M&M 5.2 335-346

Topics

- Sampling distribution of a sample mean
- Central Limit Theorem

Sampling Distribution of a Sample Mean

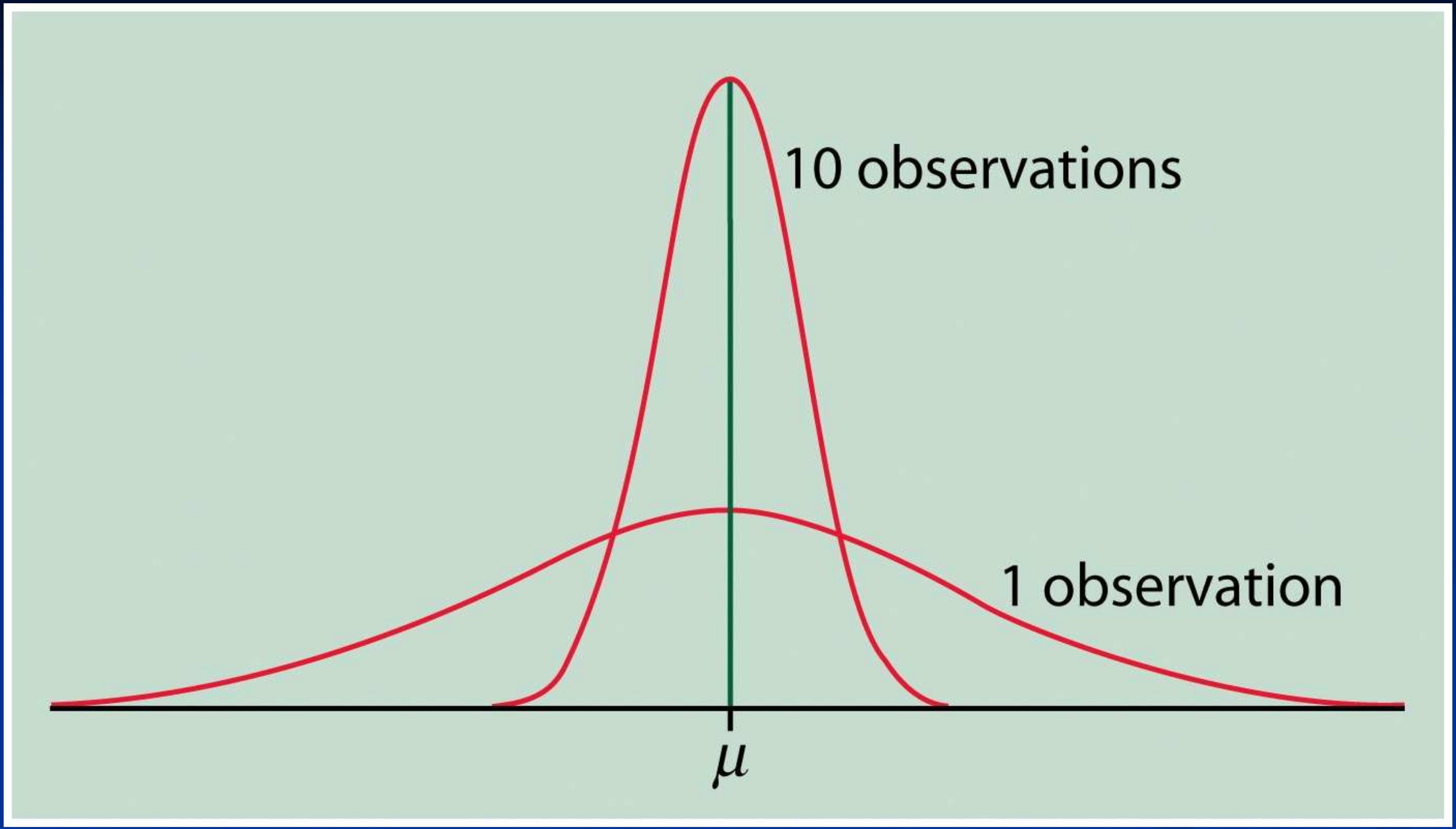
- Distribution of values taken by the sample mean in all possible samples of size **n** from the population.
- Consider a population with mean μ and standard deviation σ.
- For sample 1: SRS of size n → $\bar{x}_1$
- For sample 2: SRS of size n → $\bar{x}_2$
- For sample 3: SRS of size n → $\bar{x}_3$

Sampling Distribution of a Sample Mean

- Using simple rules for means and variances, we can show that the sample mean of a SRS of size n has a mean and standard deviation given by:

$$\mu_{\bar{X}} = \mu$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$



$$\mu_{\bar{X}} = \mu \qquad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

Sampling Distribution of a Sample Mean

- Properties of sampling distribution of a sample mean:

1) $\mu_{\bar{X}} = \mu$

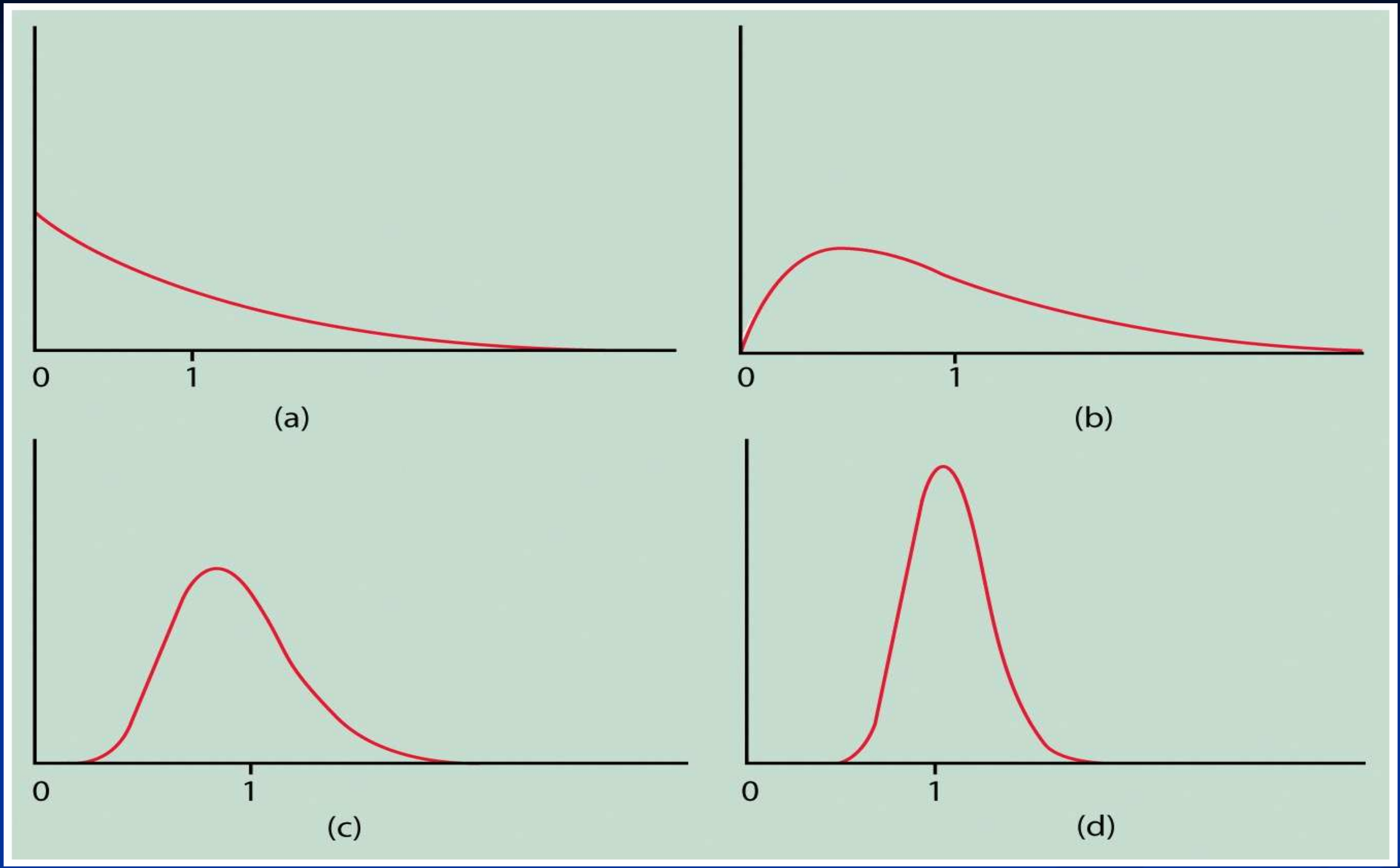
2) $\sigma_{\bar{X}} = \dfrac{\sigma}{\sqrt{n}}$

3) The distribution is normal as $n \to \infty$

The third property is due to the Central Limit Theorem.

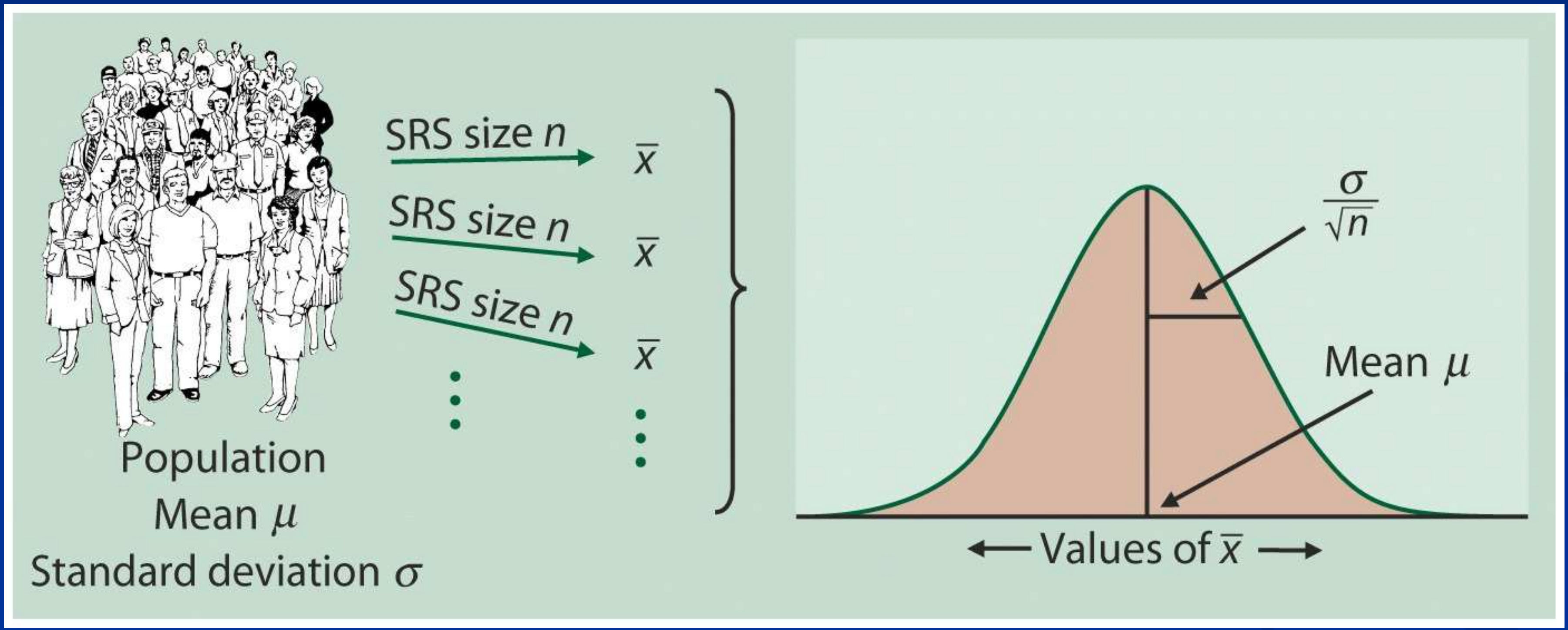
Central Limit Theorem

- Provided the sample size **n** is large enough, shape of the sampling distribution is approximately normal.
- This result applied to *any* population *regardless* of the shape of the underlying distribution.
- The farther the underlying distribution departs from a normal distribution, the larger the value of **n** necessary to ensure sampling distribution is normal.
- If underlying distribution is normal, samples of size n = 1 are large enough.



Underlying distribution is highly skewed:
(a) n=1; (b) n=2; (c) n=10; (d) n=25

Sampling Distribution of a Sample Mean



Example: Property Values

- Property values in a certain area of Maine. Let X denote property values.

  $\mu_X = \$211$ thousand

  $\sigma_X = \$46$ thousand

  Consider the sample mean of an SRS of size n=25 from this population.

Example: Property Values

- **Question**: What proportion of SRS of size 25 will have means > $230,000?

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{230 - 211}{46 / \sqrt{25}} = 2.07$$

From normal tables, area to right of 2.07 is 0.019.

Probability of obtaining a sample mean of 230 or higher (provided the true population mean is $211 thousand when taking a SRS of 25 is 0.019, or 2%