Mathematics 231

Lecture 18

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Announcements

- Reading
  - Today M&M 5.1 311-331
  - Next M&M 5.2 335-346

Topics

- Bayes Rule
- Binomial Distribution
- Sampling Distribution of a Proportion

Bayes' Rule

- The first steps in solving a nasty probability problem:
  - Define your events
  - Write down all the information you know
  - Write down what you're looking for
- Potential problem: You want P(B|A), but you have P(A|B).
- Potential solution: Bayes' Rule

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B^C)P(B^C)}$$

Bayes' Rule

- Consider a test for smoking administered to high school students
- P(T+ | Smoker) = 0.209; P(T- | Nonsmoker) = 0.967
- P(Smoker) = 0.15
- What is P(Smoker | T+)?
- Answer: 0.037

Binomial Distribution

- Let B be a dichotomous (Bernoulli) random variable with

  P(B=1) = p

  P(B=0) = 1 – p

  e.g., heads/tails, male/female, success/failure
- Consider a fixed number, **n**, of **independent** observations with constant probability of "success" (**p**) for each trial.
- Let X denote the total number of successes observed in n trials.

Binomial Distribution

- Then X has a binomial distribution, denoted X~Bin(n, p),

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{(n-x)}$$

$n =$ number of trials

x = number of successes

p = probability of success

$$\binom{\mathrm{n}}{\mathrm{x}} = \frac{n!}{x!(n-x)!}$$

Binomial Mean and SD

- If X~Bin(n, p), then the mean and SD of X are,

$$\mu_X = np$$

$$\sigma_X = \sqrt{np(1-p)}$$

- Example: If X~Bin(10, 0.5) then,

Mean = 10*0.5 = 5

SD = $(10*0.5*0.5)^{0.5}$ = 1.6

Sample Proportion

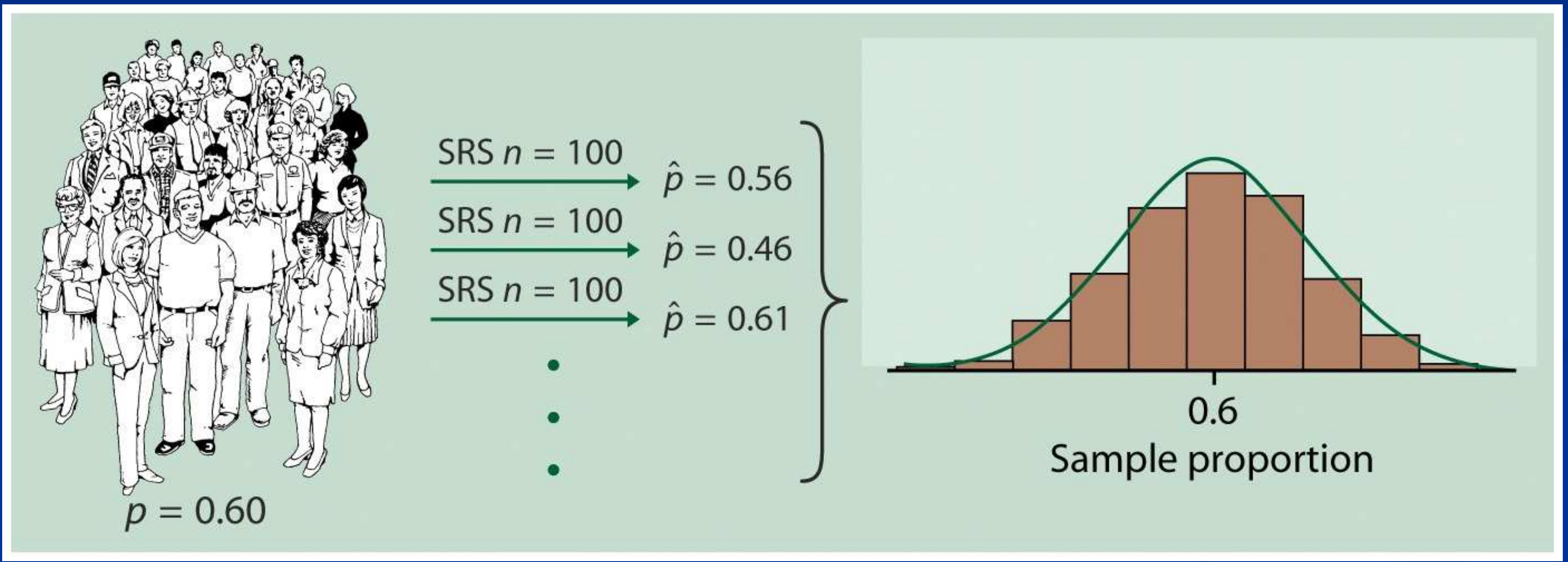
- Let X~Bin(n, p).

Consider the sample proportion, $\hat{p} = \dfrac{x}{n}$

The mean and SD of the sample proportion are,

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$


SRS n = 100
$\hat{p}$ = 0.56
SRS n = 100
$\hat{p}$ = 0.46
SRS n = 100
$\hat{p}$ = 0.61
0.6
Sample proportion
p = 0.60

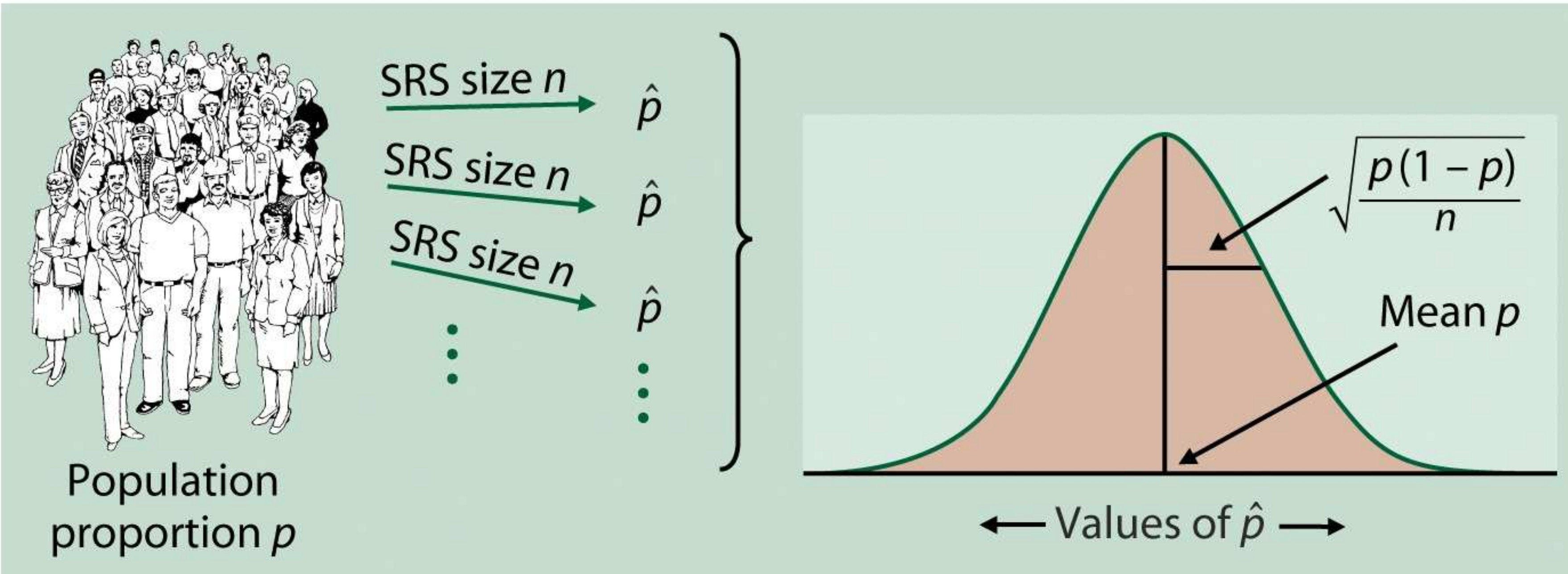
Sampling Distribution for a Proportion

- When n is large (np $\geq$ 10 and n(1-p) $\geq$ 10) then the sampling distribution of $\hat{p} = \frac{x}{n}$

  is approximately normal with

  $$\mu_{\hat{p}} = p$$

  $$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$


SRS size n
SRS size n
SRS size n
$\hat{p}$
$\hat{p}$
$\hat{p}$
Population proportion p
$\sqrt{\frac{p(1-p)}{n}}$
Mean p
Values of $\hat{p}$

ESP Experiment

- 5 cards with symbols; identify symbols with ESP
- Conducted 60,000 trials
- If no ESP, P(guess correctly) = 1/5
- Expected number correct = 60000/5 = 12,000
- Actual result: 12,489
  (sample proportion=0.20815)
- Is this result likely to have occurred by chance alone?

ESP Experiment

- When n is large (**check assumptions**),

the sampling distribution of $\hat{p} = \frac{x}{n}$

is approximately normal with

$$\mu_{\hat{p}} = p = 0.2$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.2(1-0.2)}{60000}} = .00163$$

ESP Experiment

- If there is no ESP are we likely to obtain a sample proportion of 0.20815?
- Standardize:
- Let Z = (0.20815-0.2)/0.001623 = 4.99
- P(Z > 4.99) = 0.0000003