Mathematics 231

Lecture 17

Liam O'Brien

Announcements

- Reading
  - Today M&M 4.3 258-267
    - M&M 4.4 270-286
    - M&M 4.5 289-303
  - Next class M&M 5.1 311-331

Topics

- Random Variables and Expected Values
  - Discrete random variables
  - Continuous random variables
  - Means and variances
  - Rules for means and variances

Random Variables

- **Random Variable**: A variable whose values are determined by the outcome of a random phenomenon.
- **Discrete Random Variable**: A variable having a finite number of possible values.
- If X is a discrete random variable with k possible values, its probability distribution is:

x_1	x_2	x_3	…	x_k
p_1	p_2	p_3	…	p_k

Example

- Outcome of tossing a fair coin four times.
  S = {HHHH,HHHT,HHTH,…,TTTT}

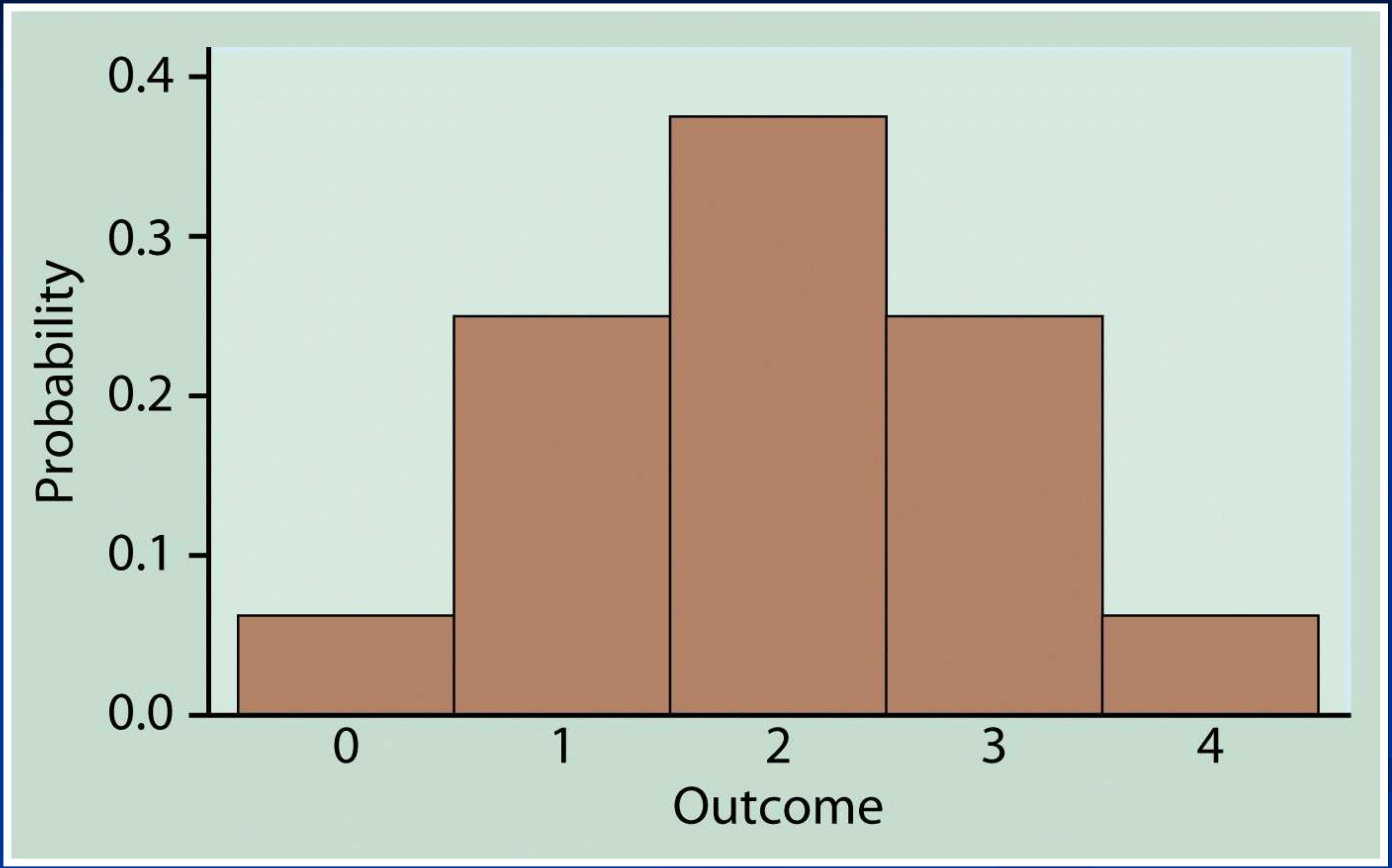
P(HHHH) = P(HHHT)=…=P(TTTT)=$(0.5)^4$=1/16

Let X be the count of number of heads in 4 tosses.

X is a discrete random variable with 5 possible values (0,1,2,3,4), its probability distribution is:

Value:	0	1	2	3	4
Probability:	1/16	4/16	6/16	4/16	1/16

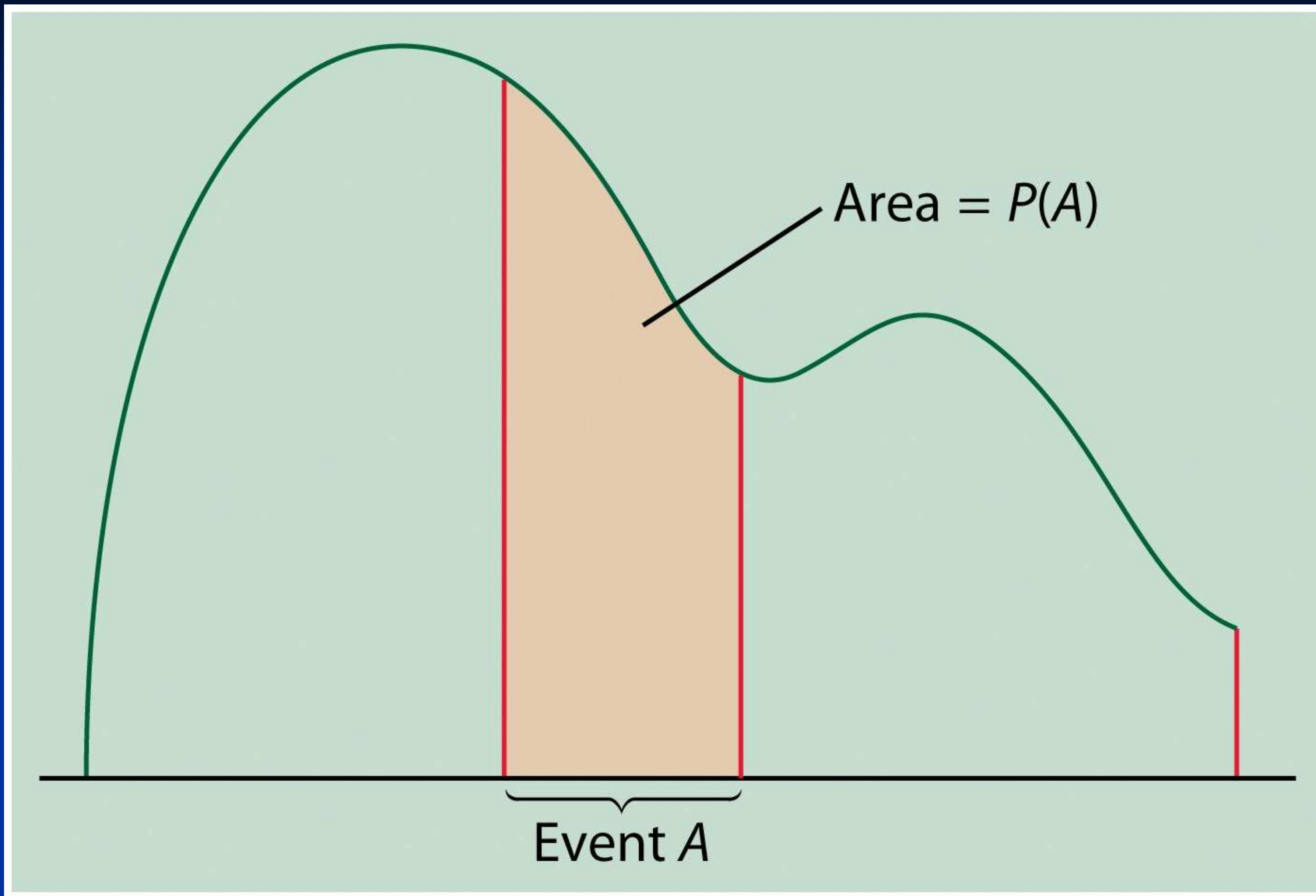
		HTTH		
		HTHT		
	HTTT	THTH	HHHT	
	THTT	HHTT	HHTH	
	TTHT	THHT	HTHH	
TTTT	TTTH	TTHH	THHH	HHHH
$X = 0$	$X = 1$	$X = 2$	$X = 3$	$X = 4$



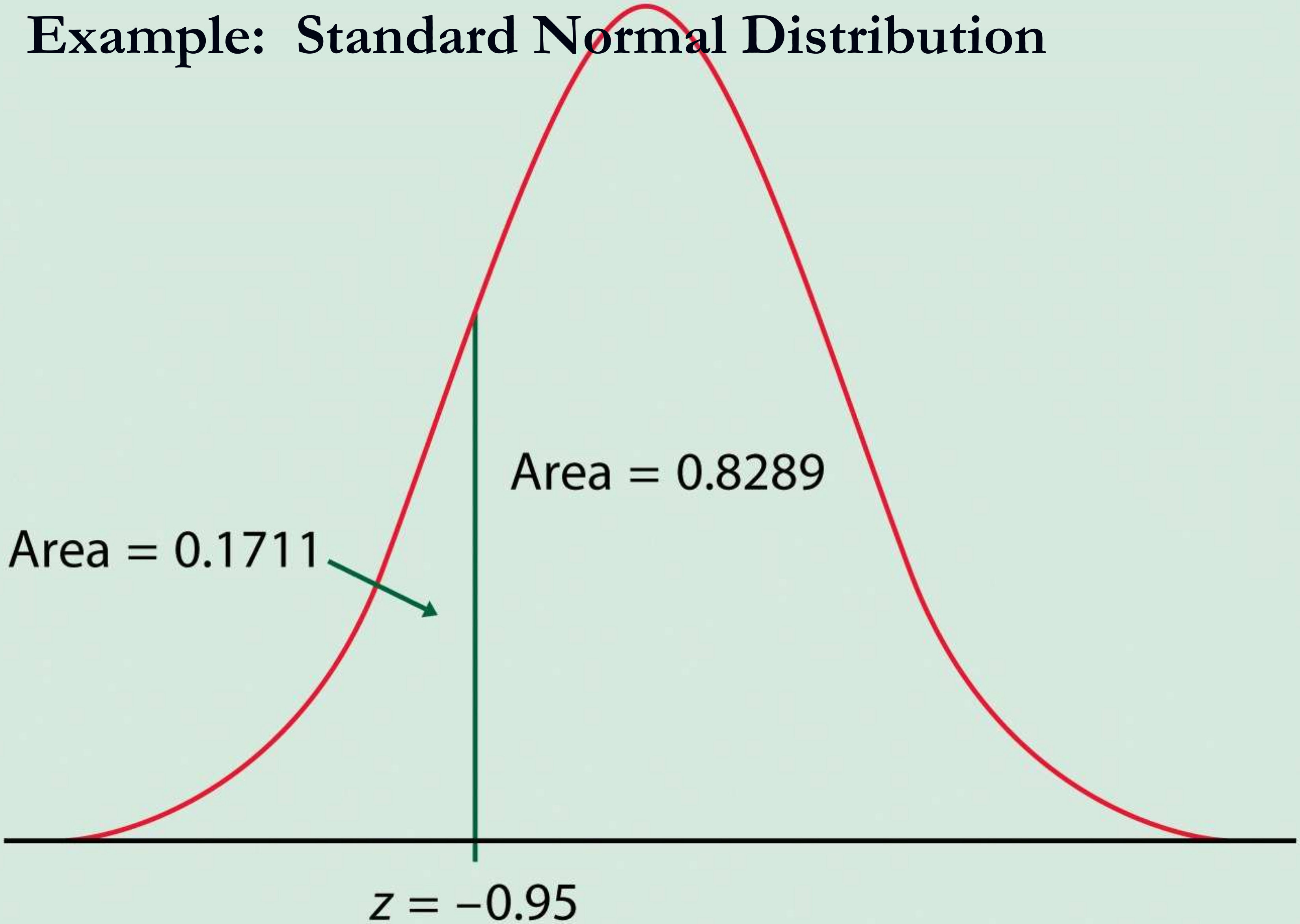
Probability
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
Outcome

Continuous R.V.

- **Continuous Random Variable:** A variable taking all possible values in an interval of numbers.
- If X is a continuous random variable, its probability distribution is described by a **density curve** and the probability of an event is the area under the curve.


Area = P(A)
Event A

Example: Standard Normal Distribution



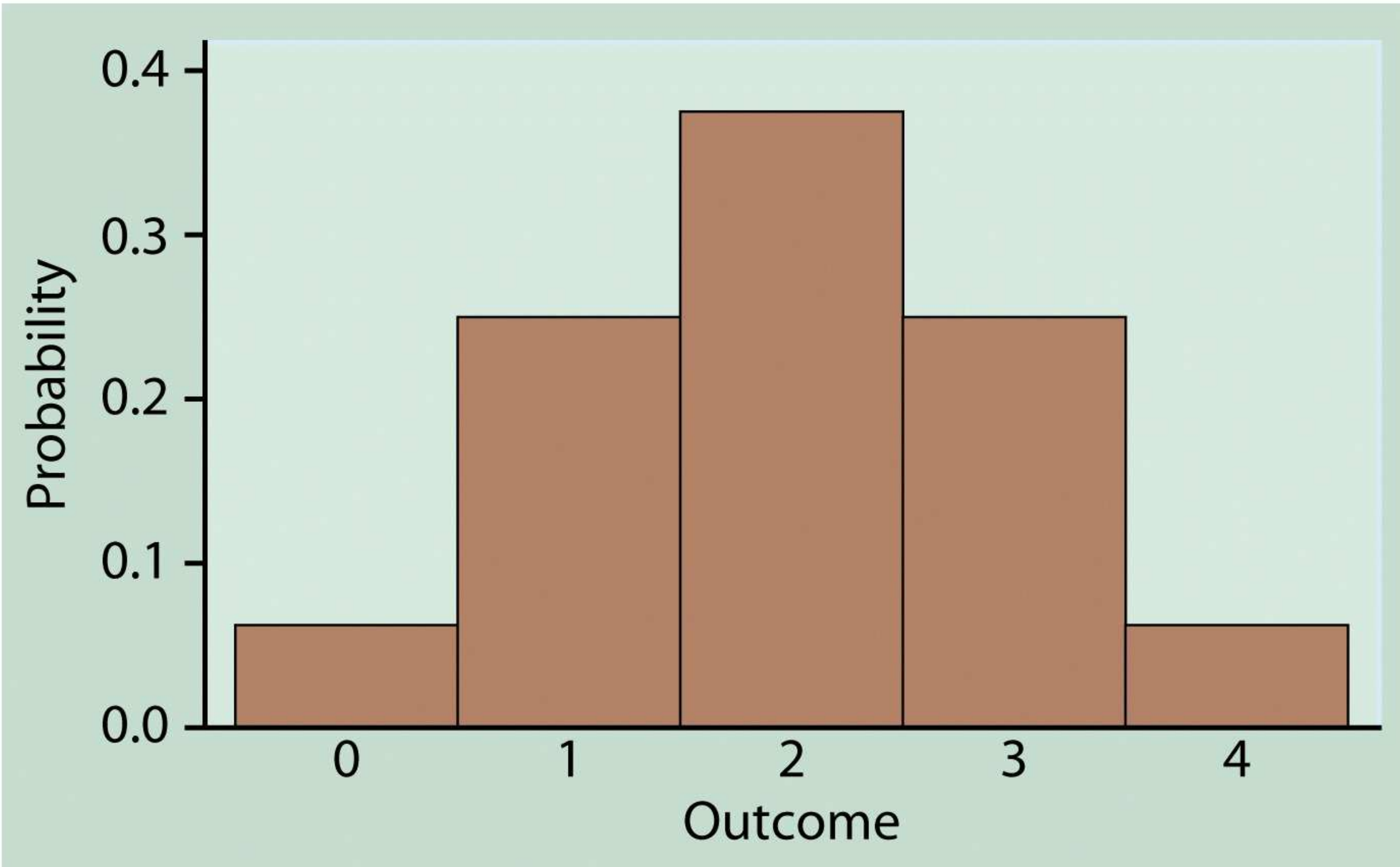
Mean of Discrete Random Variable

- If X is a discrete random variable with k possible values, its probability distribution is:

Value:	x_1	x_2	x_3	…	x_k
Probability:	p_1	p_2	p_3	…	p_k

- Mean (or "expected value") of X, denoted μ_X, is given by

$$E(X) = \mu_X = x_1 p_1 + x_2 p_2 + x_3 p_3 + \ldots + x_k p_k$$



0.4
0.3
0.2
0.1
0.0
Probability
0
1
2
3
4
Outcome

Rules for Means

- **Rule 1**: If X is a random variable, with mean μ_X and *a* and *b* are fixed constants, then the mean of $a+bX$ is

$$a+b\,\mu_X$$

- **Rule 2**: If X and Y are random variables with means μ_X and μ_Y, respectively, then the mean of X+Y is

$$\mu_X+\mu_Y$$

Variance of Discrete Random Variable

- If X is a discrete random variable with k possible values, its probability distribution is:

Value:	x_1	x_2	x_3	…	x_k
Probability:	p_1	p_2	p_3	…	p_k

- The variance of X, denoted by σ^2_X is given by

$$Var(X)= \sigma^2_X=(x_1- \mu_X)^2\, p_1+(x_2- \mu_X)^2\, p_2+\ldots+(x_k- \mu_X)^2\, p_k$$

Rules for Variances

- **Rule 1**: If X is a random variable, with variance σ^2_X, and *a* and *b* are fixed constants, then the variance of $a + bX$ is

$$b^2 \sigma^2_X$$

- **Rule 2**: If X and Y are **independent** random variables with variances σ^2_X and σ^2_Y, respectively, then the variance of X+Y or X-Y is

$$\sigma^2_X + \sigma^2_Y$$

Rules for Variances

- **Rule 3:** If X and Y have correlation, ρ, then the variance of X+Y is

$$\sigma^2_X + \sigma^2_Y + 2\rho\sigma_X\sigma_Y$$

and the variance of X-Y is

$$\sigma^2_X + \sigma^2_Y - 2\rho\sigma_X\sigma_Y$$

Notes on Variances

- Note 1: Standard deviation of X, σ_X, is simply the square-root of the variance of X.

- Note 2: If X and Y are independent, their correlation is zero ($\rho = 0$).