Mathematics 231

Lecture 16
Liam O'Brien

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Announcements

- Reading
  - Today M&M 4.0- M&M 4.2 237-254
  - Next class M&M 4.3 258-267
    M&M 4.4 270-286
    M&M 4.5 289-303

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Topics

- Probability

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Probability

- A full description of a random phenomenon requires:
  1. Listing of all the possible outcomes.
  2. List the probability associated with each outcome.
- Example: Tossing a fair coin.
  1. Possible outcomes: Heads or tails
  2. Probability of heads = 0.5; probability of tail = 0.5

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Sample Space

- **Sample Space**: Set of all possible outcomes, usually denoted by **S**.
- Example: Tossing a fair coin

  S = {Heads, Tails}
- Example 2: Tossing a fair coin three times and counting the number of heads.

  S = {0,1,2,3}

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Sample Space and Events

- **Event**: An event is an outcome or a set of outcomes and is a subset of the sample space S.
- Events (often denoted with the letters A, B, C,…) have associated probabilities.
- Example: Outcome of tossing a fair coin three times:

  S = {HHH,HHT,HTH,HTT,THH,THT,TTH,TTT}
- Let A denote the event of 2 heads;

  A = {HHT,HTH,THH}

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Probability

- The **probability** if an event is its long-term relative frequency of occurrence.
- If an experiment is repeated **n** times under essentially identical conditions and the event **A** occurs **m** times, then as n gets large,

  P(A) = m/n

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Probability

- An event that always occurs: n/n = 1

  P("sure thing") = 1
- An event that never occurs: 0/n = 0

  P("impossible") = 0

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Probability

- For any event A,
  $m \leq n$, so
  $0 \leq P(A) \leq 1$
- Complement
  $P(A) = m/n$
  $P(A^c) = (n-m)/n = 1 - P(A)$
  $P(A) + P(A^c) = 1$

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Disjoint or Mutually Exclusive Events

- Any two events A and B are **disjoint** if they have no outcomes in common and so cannot ever occur simultaneously.
- If A and B are disjoint:
  P(A or B) = P(A) + P(B)
  This is the addition rule for disjoint events.

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Overlapping Events

- If two events A and B are not disjoint, they have outcomes in common and the addition rule cannot be applied.
- Instead:
  P(A or B) = P(A) + P(B) – P(A and B)

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Conditional Probability

- Conditional probability is the probability of B **given**, or knowing that the event A has happened.

  $P(B|A) = P(A \text{ and } B)/P(A)$

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General Multiplication Rule

- P(A and B) = P(A) P(B|A)
- Note:
  - P(B|A) = P(A and B)/P(A)
  - P(A|B) = P(A and B)/P(B)
- P(A and B) = P(A) P(B|A)
  = P(B) P(A|B)

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Multiplication Rule for Independent Events

- A and B are said to be "independent" if
  P(A and B) = P(A) P(B)
- Also, since in general:
  P(A and B) = P(A) P(B|A)
  independence implies that P(B|A) = P(B).
- Similarly, independence implies that
  P(A|B) = P(A)

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Example

- A mandatory drug test has a false-positive rate of 1.2% (or 0.012).
- Given 150 employees who are drug free, what is the probability that at least one will (falsely) test positive?
- P(at least one positive)=P(1 or 2 or 3…or 150 positive)
  = 1 – P(none positive)
  = 1 – P(150 negative)

P(150 negative) = $(0.988)^{150}$ = 0.16

P(at least 1 positive) = 0.84

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Summary

1. For any event A, $0 \leq P(A) \leq 1$
2. P(S) = 1, where S is the sample space
3. For any event A, $P(A^c) = 1 - P(A)$
4. Addition Rule: P(A or B) = P(A) + P(B) – P(A and B)
5. Conditional Probability: P(B|A)=P(A and B)/P(A)
6. Multiplication Rule for Independent Events:
   If A and B are independent, P(A and B) = P(A)P(B)

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Why Intuition Can Be BAD

- Using intuition when it comes to probability has caused many people some BIG problems.
- Consider the Gambler's Fallacy:
  - A gambler feels like he/she is on "a roll." The gambler continues to wager more and more money assuming that the streak of luck will continue.
  - Ultimately, the gambler will lose.
- Law of small numbers:
  - Small samples do not represent the population well. Thus, laws of probability may not appear to hold.

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Why Intuition Can Be BAD

- Confusion of the inverse:
  - The Eddy Scenario (1982).
  - Doctors asked if a patient has a positive mammogram, and the test is 90% for those with benign lumps (and 80% accurate for those with malignancies), what is the probability the patient has breast cancer?
  - Prevalence of breast cancer is 0.01.
  - Most doctors said 0.75.
  - In truth, it's 0.075.

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Why Intuition Can Be BAD

- The Birthday Problem
  - If there are 27 of you, what is the probability that at least two of you share the same birthday? 0.627
  - In a class of 50, the probability is 0.970.
  - These numbers seem counterintuitive.
- How can we calculate this?

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