Mathematics 231

Lecture 15

Announcements

- Reading
  - Today   Rest of Chapter 3
  - Next class   M&M 4.0-   237-254
              M&M 4.2

Sampling Design

- In sample surveys we want to obtain information from a part of a group to draw conclusions about the whole group.
- Population → Sample
  - **Population**: Entire group of individuals we desire information on.
  - **Sample**: Part of population we actually collect data from.
  - **Sampling Design**: Method used to choose sample from population.

Parameter and Statistic

- **Parameter**: Number that describes the population.
- **Statistic**: Number that describes a sample.

- We use a statistic to estimate an unknown parameter.

Simple Random Survey (SRS)

- In an SRS of size *n*:
1. Each individual has an *equal* chance of being chosen.
2. Every set of *n* individuals has an equal chance of being the sample chosen.

Stratified Samples

- **Basic Idea**: Sample important groups separately, then combine those samples.
1. Divide population into groups of similar individuals, called **strata**.
2. Choose a separate SRS within each strata.
3. Combine these SRS's to form the full sample.

Stratified Samples

- Strata for sampling are similar to blocks in experiments.
- Stratified sampling designs can provide more precise information than an SRS of the same size.
- For example, if all individuals within each stratum are identical, only need one individual from each stratum to perfectly describe the population.

Multistage Samples

- Basic Idea: Choose sample in stages.

- Often used for national surveys (U.S. households).
- Not practical to do SRS from list of all U.S. households (cost, inconvenience, time).

Multistage Samples

- To take a nationwide multistage sample:
1. Take sample from the 3000 counties in the U.S.
2. Take a sample of townships within each county chosen.
3. Take a sample of city blocks within each township chosen.
4. Take a sample of households within each city block.
- At each stage, take random sample (e.g., an SRS)

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Pitfalls of Sample Surveys

- **Selection Bias**: Some groups in population are over- or under-represented in sample.
- **Nonresponse Bias**: Nonrespondents may differ in important ways from respondents.
- **Response Bias**: e.g., wording of question, ordering of questions, telescoping in the recall of events.

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1936 Literary Digest Poll

- Literary Digest had predicted the winner of every U.S. presidential election since 1916.
- In 1936, Literary Digest mailed questionnaires to 10 million people (25% of voters).
- 2.4 million people responded, the largest number of people ever replying to a poll.
- **Prediction**:   Roosevelt 43%, Landon 57%
- **Actual Result**: Roosevelt 62%, Landon 38%

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Selection and Nonresponse Bias

- **Selection Bias**: People surveyed came from telephone books, club memberships, mail order lists, automobile ownership lists.
- **Nonresponse Bias**: 76% did not respond.

- The Gallup Poll predicted Roosevelt's victory with a sample of 50,000 people.

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Response Bias

- Wording of question can deliberately bias:
  - Do you favor, or do you not favor, increased restrictions on public smoking?
  - Do you favor Gestapo-like police tactics to prevent smoking in public?
  - Do you think smokers have the right to impose their filthy habits on the rest of us, polluting our precious air?

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Response Bias

- Social Desirability:
  - Surveys of smoking underestimate the prevalence of smoking and do not match cigarette sales.
- Uninformed:
  - Survey by the American Jewish Committee on attitudes toward various ethnic groups.
  - "30% of respondents expressed an opinion about the Wisians…"

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Statistical Inference

- Basic Idea: Use sample (statistic) estimate to infer conclusion about the population (parameter).
- Need to distinguish between sample and population values (statistics vs. parameters).
- Parameters are numbers that describe (unknown) characteristics of the population.
- Statistics are numbers that describe a sample.

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Statistical Inference

- Statistics: Numbers that describe a sample.
- We use statistics to estimate unknown parameters.
- Although a statistic is known once we have selected our sample, it can change from sample to sample.
- The is referred to as **sampling variability**.

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Sampling Distribution

- Question: What would happen if the sample or experiment were repeated many times?
- Consider the following "thought experiment":
- Take repeated samples of the same size from the same population.
  - 1st sample, calculate the statistic of interest
  - 2nd sample, calculate the statistic of interest
  - 3rd sample, calculate the statistic of interest
   and so on…

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Sampling Distribution

- The statistic will vary from sample to sample due to **sampling variability**.
- **Sampling distribution** of a **statistic** is the distribution of values taken by the statistic in all possible samples of the same size from the same population.
- The sampling variation has a predictable pattern.

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Sampling Distribution

- Example: Opinion Poll
1. Take a large number of samples of size n (e.g., n = 100) from the population.
2. Calculate the sample statistics for each sample (e.g., the proportion of people supporting Bush).
3. Make a histogram of the sample proportions.
4. Examine the distribution and determine center, spread, and shape.

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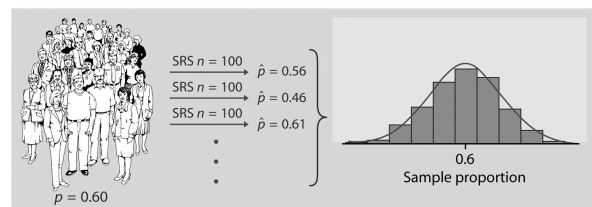
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Sampling Distribution

- **Center**: Values are centered at the true population parameter.
- **Spread**: Samples of size 1000 are much less variable than samples of size 100.
- **Shape**: Sampling distribution is approximately normal under certain conditions. Moreover, approximation gets better as the sample size, n, gets larger.
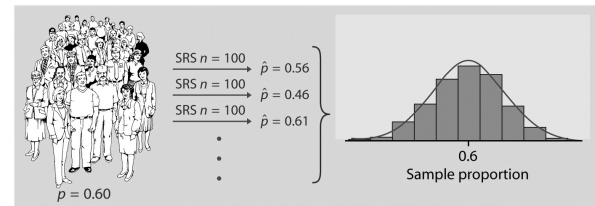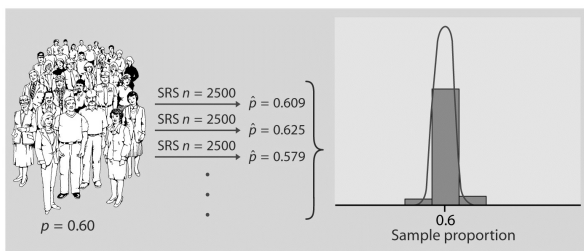
Simplified Example

- Partying habits of statistics students at one of those inferior schools (e.g., Bates).
- Suppose there are only 5 statistics students who went to Westminster High School, then the entire population consists of these 5 students:

  Name: Jerry Mary Gary Larry Sue
  Parties: 2   4   4   6   8

Simplified Example

- Take an SRS of two students from this population.

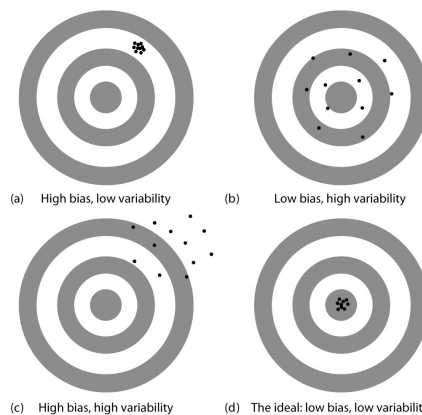| Names	# Parties	Mean
Jerry, Mary	2,4	3
Jerry, Gary	2,4	3
Jerry, Larry	2,6	4
Jerry, Sue	2,8	5
Mary, Gary	4,4	4
Mary, Larry	4,6	5
Mary, Sue	4,8	6
Gary, Larry	4,6	5
Gary, Sue	4,8	6
Larry, Sue	6,8	7

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Bias and Variability

- **Bias**: Concerns the center of the sampling distribution.
- A statistic is said to be **unbiased** if the mean of its sampling distribution is equal to the true value of the parameter.
- Bias is reduced by using random sampling.
- If randomization of not done properly, then bias can be introduced. This is BAD.
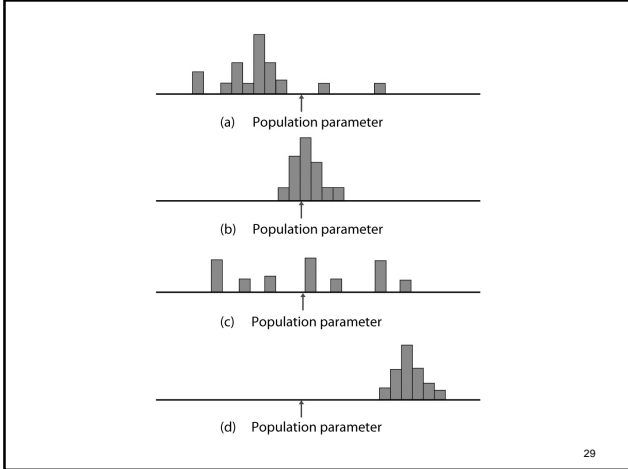
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Bias and Variability

- The variability of a statistic is described by the spread of its sampling distribution.
- Variability is reduced by using a larger sample size, n.
- Results of a sample survey usually come with a **margin of error**.
- This sets bounds on the size of the likely error.

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(a) High bias, low variability   (b) Low bias, high variability

(c) High bias, high variability   (d) The ideal: low bias, low variability

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(a) Population parameter

(b) Population parameter

(c) Population parameter

(d) Population parameter

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Population Size Doesn't Matter

- **Population size doesn't matter**.
- The variability of a statistic from a random sample does not depend on the size of the population (provided the population is substantially larger than the sample).
- Important consequences for surveys:
  - An SRS of 2500 from the more than 210 million adults in U.S. gives results as precise as an SRS of 2500 from the 665,000 inhabitants of San Francisco.

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Population Size Doesn't Matter

- Intuition:
  - Imagine you're a chef tasting soup. As long as the soup is well mixed (ensuring a random sample), the variability of the results depends only on the size of the spoon (sample) and not on the size of the pot (population).

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