MA231 – Spring 2009 **Homework 6** **Due 3/18/2009**

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Homework Policies: You should give a brief and concise explanation for each question. Just writing down an answer with no explanation is usually not sufficient. If the homework requires output from Stata, incorporate that output into your written assignments. Homework is due at the *beginning* of class on the day indicated.

(1) M & M 4.21, p. 255

4.21. **(a)** The given probabilities have sum 0.55, so $P(\text{type O}) = 0.45$. **(b)** $P(\text{type O or B}) = 0.45 + 0.11 = 0.56$.

(2) M & M 4.29, p. 256

4.29. For example, the probability for O-positive blood is $(0.45)(0.84) = 0.378$ and for O-negative $(0.45)(0.16) = 0.072$.

Blood type	O+	O–	A+	A–	B+	B–	AB+	AB–
Probability	0.3780	0.0720	0.3360	0.0640	0.0924	0.0176	0.0336	0.0064

(3) M & M 4.39, p. 257-258

4.39. **(a)** $(0.65)^3 \doteq 0.2746$ (under the random walk theory). **(b)** 0.35 (because performance in separate years is independent). **(c)** $(0.65)^2 + (0.35)^2 = 0.545$.

(4) M & M 4.60, p. 269

4.60. Let "S" mean that a student supports funding and "O" mean that the student opposes funding. **(a)** $P(\text{SSO}) = (0.6)(0.6)(0.4) = 0.144$. **(b)** See the top two lines of the table below. **(c)** The distribution is given in the bottom two lines of the table. For example, $P(X = 0) = (0.6)(0.6)(0.6) = 0.216$, and in the same way, $P(X = 3) = 0.4^3 = 0.064$. For $P(X = 1)$, note that each of the three arrangements that give $X = 1$ have probability 0.144, so $P(X = 1) = 3(0.144) = 0.432$. Similarly, $P(X = 2) = 3(0.6)(0.4)(0.4) = 0.288$. **(d)** A majority means $X \geq 2$; $P(X \geq 2) = 0.288 + 0.064 = 0.352$.

Arrangement	SSS	SSO	SOS	OSS	OOS	OSO	SOO	OOO
Probability	0.216	0.144	0.144	0.144	0.096	0.096	0.096	0.064
Value of X	0		1			2		3
Probability	0.216		0.432			0.288		0.064

(5) M & M 4.74, p. 286-287

4.74. The mean number of nonword errors is $(0)(0.1)+(1)(0.3)+(2)(0.3)+(3)(0.2)+(4)(0.1) = 1.9$, and the mean number of word errors is $(0)(0.4) + (1)(0.3) + (2)(0.2) + (3)(0.1) = 1$.

(6) M & M 4.78, p. 287

4.78. Let N and W be nonword and word error counts. In Exercise 4.74, we found $\mu_N = 1.9$ errors and $\mu_W = 1$ error. The variances of these distributions are $\sigma_N^2 = 1.29$ and $\sigma_W^2 = 1$, so the standard deviations are $\sigma_N \doteq 1.1358$ and $\sigma_W = 1$ errors. The mean total error count is $\mu_N + \mu_W = 2.9$ errors for both cases. **(a)** If error counts are independent, $\sigma_{N+W}^2 = \sigma_N^2 + \sigma_W^2 = 2.29$ and $\sigma_{N+W} \doteq 1.5133$ errors. (Note we add the *variances,* not the standard deviations.) **(b)** With $\rho = 0.4$, $\sigma_{N+W}^2 = \sigma_N^2 + \sigma_W^2 + 2\rho\sigma_N\sigma_W \doteq 2.29 + 0.9086 = 3.1986$ and $\sigma_{N+W} \doteq 1.7885$ errors.

(7) M & M 4.91, p. 289

4.91. With $R = 0.8W + 0.2Y$, we have $\mu_R = 0.8\mu_W + 0.2\mu_Y = 11.116\%$ and:

$$\sigma_R = \sqrt{(0.8\sigma_W)^2 + (0.2\sigma_Y)^2 + 2\rho_{WY}(0.8\sigma_W)(0.2\sigma_Y)} \doteq 15.9291\%$$

(8) M&M 4.92, p. 289

4.92. With $\rho_{WY} = 0$, the standard deviation drops to $\sqrt{(0.8\sigma_W)^2 + (0.2\sigma_Y)^2} \doteq 14.3131\%$. The mean is unaffected by the correlation.

(9) M&M 4.127, p. 306

4.127. Let T be the event "test is positive" and C be the event "Jason is a carrier." Since the given information says that the test is never positive for noncarriers, it clearly must be the case that $P(C \mid T) = 1$.

To confirm this, note that (if there is no human error) we have $P(T \text{ and } C^c) = 0$ and $P(T) = P(T \text{ and } C) + P(T \text{ and } C^c) = P(T \text{ and } C) = P(T)\,P(T \mid C) = (0.04)(0.9) = 0.036$. Therefore, $P(C \mid T) = \frac{P(C \text{ and } T)}{P(T)} = \frac{0.036}{0.036} = 1$.

(10) M&M 4.128, p. 306

4.128. Let C be the event that Julianne is a carrier, and let D be the event that Jason's and Julianne's child has the disease. We have been given $P(C) = \frac{2}{3}$, $P(D \mid C) = \frac{1}{4}$, and $P(D \mid C^c) = 0$. Therefore, $P(D^c) = P(C)\,P(D^c \mid C) + P(C^c)\,P(D^c \mid C^c) = \left(\frac{2}{3}\right)\left(\frac{3}{4}\right) + \left(\frac{1}{3}\right)(1) = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$, and $P(C \mid D^c) = \frac{1/2}{5/6} = \frac{3}{5}$.

(11) M&M 5.14, p. 332

5.14. **(a)** X, the number of auction site visitors, is $B(15, 0.5)$. **(b)** Symmetry tells us that this must be 0.5, which is confirmed in Table C: $P(X \geq 8) = 0.1964 + 0.1527 + 0.0916 + 0.0417 + 0.0139 + 0.0032 + 0.0005 = 0.5$.

(12) M&M 5.48, p. 347

5.48. **(a)** $P(X \geq 23) \doteq P(Z \geq \frac{23-20.8}{4.8}) = P(Z \geq 0.46) = 0.3428$ (with software: 0.3234). Because ACT scores are reported as whole numbers, we might instead compute $P(X \geq 22.5) \doteq P(Z \geq 0.35) = 0.3632$ (software: 0.3616). **(b)** $\mu_{\bar{x}} = 20.8$ and $\sigma_{\bar{x}} = \sigma/\sqrt{25} = 0.96$. **(c)** $P(\bar{x} \geq 23) \doteq P(Z \geq \frac{23-20.8}{0.96}) = P(Z \geq 2.29) = 0.0110$. (In this case, it is not appropriate to find $P(\bar{x} \geq 22.5)$, unless $\bar{x}$ is rounded to the nearest whole number.) **(d)** Because individual scores are only roughly Normal, the answer to (a) is approximate. The answer to (c) is also approximate but should be more accurate because $\bar{x}$ should have a distribution that is closer to Normal.