Math 132 - Topology II: Smooth Manifolds. Harvard University. Spring 2017, MWF 1-2pm. Science Center 310.

Instructor: George Melvin
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Office: Science Center 209h
Office hours: Monday, 11am-12pm. Thursday 4.30-5.30pm.
Course website: http://math.harvard.edu/~gmelvin/current.html

Announcements and handouts can be found at the website; please check frequently.

Course assistant: Julian Salazar (jsalazar01@college.harvard.edu). The course assistant will hold a weekly one hour *triage* session to discuss homework and other topics of concern. The course assistant will also schedule a weekly office hour. Details forthcoming.

Course description: A (Euclidean) manifold is a geometric object that locally looks like Euclidean n-space. Smooth manifolds are manifolds together with additional structure that allows us to apply the methods of calculus. To a mathematician, the study of smooth manifolds is the branch of mathematics known as *differential geometry* and lies at the crossroads of geometry, analysis and partial differential equations (and beyond).

In this course, we will use the methods of differential geometry to understand the *topology* of a geometric object: we will relate infinitesimal information with global topological information. This is the realm of *differential topology* and has produced some of the most beautiful, deep and astounding mathematics.

Topics: Smooth manifolds & smooth maps, manifolds with boundary; derivatives and tangent spaces; immersions, submersions, embeddings; homotopy, stability; Morse functions. Transversality & (oriented-)intersection theory(-mod 2); degree theory; Lefschetz fixed point theory. Highlights: Whitney embedding theorem; Jordan-Brouwer separation theorem; Borsuk-Ulam theorem; Poincaré-Hopf theorem; Hopf degree theorem; exterior calculus, integration and generalised Stokes' theorem.

Learning outcomes & objectives: Gain exposure and understand the basic definitions and theorems pertaining to smooth manifolds and smooth maps, (oriented-)intersection theory(-mod 2), degree theory, Lefschetz fixed point theory. Build an appreciation for the types of problems appearing in differential topology. Learn computational techniques and apply them in particular examples. Gain experience writing mathematical papers and proficiency using LaTeX.

Prerequisites: Undergraduate linear algebra and real analysis, multivariable calculus. In particular, the notions of open/closed subsets of Euclidean n-space, connectedness, compactness should be familiar. Some basic group theory would be advantageous, though not necessary.

Typically, Math 23/25/55 a,b or Math 112 are the required prerequisites. Students who have not taken any of these courses should contact me immediately.

Math 131 is not a prerequisite to take Math 132.

Resources:
Course textbook:

Differential Topology, Guillemin & Pollack, AMS Chelsea Pub. (2010 Edition)

Supplementary texts:

Calculus on Manifolds, Spivak

Topology from the Differentiable Viewpoint, Milnor

Differential Topology, Hirsch

The above texts will be held on reserve in the Cabot Science library. Further references will be specified at the course website.

Grading: Homework 35%, Project I 25%, Project II 40%.

Grading policy: If either Project I or Project II is not submitted then you will fail the class. If there is a problem submitting either project you must let me know immediately.

Diagnostic quiz: There will be a *30-minute diagnostic quiz* on **2/15**. This quiz will **not** count for credit. Its intention is to allow for student-instructor engagement and feedback. The quiz will consist of demonstrating knowledge of basic definitions and examples; there will be no proofs.

Homework: Homework will be due every Wednesday (beginning Wednesday 2/1, with the exception of Wednesday 3/15 during Spring Recess). A hardcopy must be submitted in class by 1.15pm. **Late homework will not be accepted**. If you are unable to submit homework, due to unforeseen exceptional circumstances, then contact me immediately.

Homework should be clearly legible and written in complete English sentences.

Late days: You will be permitted six *late days* that you can use throughout the semester: for example, handing in homework late on the immediately subsequent Friday (before 5pm) will use up two late days. You must email me if you intend to make use of this policy.

Homework collaboration policy: You are encouraged to collaborate on solutions to problem sets with your peers. However, you must write up solutions *on your own*. Any collaboration undertaken to obtain a solution must be explicitly acknowledged.

You should not make use of online forums (eg `math.stackexchange.com`) to complete or check your solutions: consulting such forums is not considered *collaboration* in this class. You are reminded of your commitment to the Harvard College Honor Code.

Take-home projects: Part of your grade will be the submission of two take-home projects, Project I and Project II. Each project will provide you with an opportunity to write a lecture for your classmates on a topic related to the material we have discussed in class. The project will be a 5-8 page paper and must be written up using LaTeX. LaTeX templates and examples will be made available. I will provide a list of project topics and guidelines in advance. Individual project proposals are warmly welcomed, although such a proposal must be authorised by me.

You **will not** be required to present your paper to the class.

The submission dates are:

Project I: Friday 3/10, 1pm.

Project II: Thursday 5/4, 12pm.

You will have an opportunity to submit a draft copy to me several days prior to the hard submission deadline so I can provide feedback/suggestions. **Late projects will not be accepted**. **Projects I & II must be submitted or you will fail the course**. If you are unable to submit a project, due to unforeseen exceptional circumstances, then contact me immediately.

Project collaboration policy: The paper should be your own work. You are not permitted to collaborate with others. However, you are free to consult experts and any sort of reference material. Such consultations and references must be explicitly acknowledged in your paper.

Accommodations: Students with an accommodations letter from the AEO should provide these to me as soon as possible.

Important dates:

2/15: Diagonostic quiz
2/20: University holiday, no class.
2/21: Fifth 'Monday'
3/10: Project I submission
3/11-3/19: Spring Recess
4/26: Last day of instruction
5/4: Project II submission